A Demonstration of Magnification of Dynamic Topography at the Thermocline[1]

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Since the ocean is hydrostatic to a very good approximation, it is well known that dynamic topography at the surface (for example, across the Gulf Stream) is inverted and magnified at the thermocline by a factor proportional to the average density difference of the regions above and below the thermocline. This effect, also observable in the atmosphere, was first explained by Margules (1906). This effect is a maximum if the velocities in the lower layer are zero. If $h(x)$ is the surface topography, then the interface will vary as $H(x) = [\varrho_1/(\varrho_2-\varrho_1)]\,h(x)$; see Fig. 1.

An idealized system that illustrates this interesting inverse magnification can be constructed in the laboratory by utilizing a cylindrical tank, of radius r, that contains two immiscible fluids of depths H_1 and H_2 and of slightly differing densities. The tank is set into rotation with angular velocity Ω. The following sequence of events occurs: the lower fluid, being in contact with a rigid surface (the bottom), comes into rigid-body motion through Ekman layer suction (Greenspan and Howard 1963) in a time proportional to $H/(\Omega\nu)^{1/2}$, where H is the thickness of the lower fluid, Ω is the rotation rate, and ν is the viscosity. The upper fluid, however, will *not* come into rigid-body motion as rapidly as this, because the effects of the side walls and the lower fluid boundary are not as efficient in producing the secondary flows necessary for establishment of the new rigid rotation. The upper fluid will spin up in a time that is intermediate between the diffusive time,

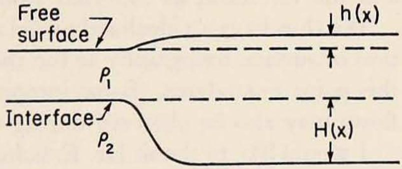

Figure 1. Inverse magnification of the surface topography over the layer of no motion.

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r^2/ν (due to diffusion from the side walls), and the spin-up time, $H/(\Omega\nu)^{1/2}$. Before both layers have spun up, however, there will be a critical time during which the lower fluid is spinning rapidly and the upper fluid is rotating very slowly, if at all. Typical values for such a laboratory experiment are $\Omega = 9.0$ rad/sec, $H_1 = H_2 = 6.4$ cm, $r = 6.9$ cm, $\varrho_1 = 0.792$ g/cc (methanol), and $\varrho_2 = 0.776$ g/cc (thin-ex, a commercially available paint thinner). This critical time occurs at about two minutes after the rotation is initiated.

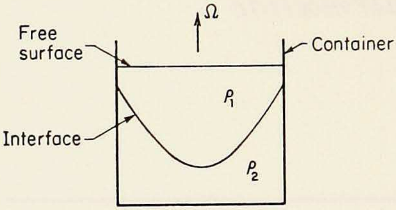

Figure 2. Observed interface displacement after the lower fluid has reached rigid-body motion and the upper fluid is still relatively motionless in the rest frame.

At this time, a remarkable phenomenon is observed: a greatly magnified parabola at the interface and a level upper surface (Fig. 2). If we imagine ourselves in the rotating system, we can understand this phenomenon from the point of view of a layer of no motion underlying a moving layer—a typical oceanic situation. In the rotating system, the lower layer will have zero *relative* motion while the upper layer (motionless in the rest frame) will appear to be rotating in the opposite direction when observed from the rotating frame. The upper surface, though level in the rest frame, will not be level in the rotating frame. Since a level surface in the rotating frame is paraboloid, a surface that is level in the rest frame will appear dome-shaped to the observer in the rotating frame. This dome-shaped surface will be inverted and magnified at the interface by the density difference ratio, as indicated above, thus producing the large parabola. When the upper fluid finally spins up, we observe gently sloping parabolas at both the surface and the interface, as expected from rigid-body rotation in this system.

We thus have an idealized model and demonstration of the inverse magnification of surface topography at the thermocline; this can be useful in illustrating this point to students. (Some interesting and as yet unexplained nonsymmetric flows may also be observed during this transient flow.)

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