Oceanic Leveling by a Vessel Crossing a Current[1]

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When a current is crossed by a vessel on fixed course at fixed speed, the vessel is displaced downstream. The displacement, D, is proportional to the current speed integrated across the width of the current and is inversely proportional to the vessel's speed. If the current is geostrophically balanced, the integrated current speed is directly related to the difference in level across the current. The displacement D can be measured under some conditions, and this measurement is proposed as a method for determining the difference in level across relatively swift ocean currents. The method is a special application of the geostrophic equation, which is often used to calculate currents from isobaric topography and sometimes to calculate isobaric topography from currents.

Assume a straight parallel current flowing with speed v in the y-direction, normal to the x-direction. Let S be the x-component of the vessel's velocity through the water. If t is time during the vessel's travel,

$$\mathrm{d}x = S\,\mathrm{d}t,$$

$$D = \int v\,\mathrm{d}t = S^{-1}\int v\,\mathrm{d}x.$$

For geostrophic balance,

$$fv = (\mathrm{d}\varphi/\mathrm{d}x)_p,$$

where f is the Coriolis parameter, and the right side is the cross-stream geopotential slope of an isobaric surface. The geopotential difference across the current is

$$\Delta\varphi = \int fv\,\mathrm{d}x.$$

Neglecting the variation in f and substituting the equation for D leads to the simple result that

$$\Delta\varphi = DSf.$$

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For a Gulf Stream crossing, the orders of magnitude are as follows:

$$\Delta\varphi = 10\ \mathrm{J/kg} = 10\ \mathrm{m^2/s^2},$$
$$f = 10^{-4}\ \mathrm{s^{-1}},$$
$$S = 5\ \mathrm{m/s},$$
$$D = 2 \times 10^4\ \mathrm{m} = 20\ \mathrm{km}.$$

The displacement D must be measured as the difference between a navigational fix and a position from dead reckoning. The error in fix depends on the navigational system available. To get an idea of the error in dead reckoning, suppose an error in measured course of 1° in a crossing 200 km long (11 hours); the resulting error in displacement is 3.5 km or 18%.

The proposed method is more reliable in weak than in strong winds. Wind produces a drift current that is superimposed on the geostrophic current; the drift current, having a surface speed of roughly 2% of the wind speed, contributes to the vessel's displacement but not to the slope of the sea surface. Furthermore, wind may cause leeway of the vessel, the sea accompanying strong wind increases the error in course, and wind and sea decrease the accuracy of the vessel's speed.