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Recirculation reconsidered

by James Luyten and Henry Stommel

ABSTRACT

The Atlantis 50W IGY section is used to test ideas about the application of inverse methods to the determination of the absolute geostrophic circulation of the North Atlantic. We argue that the uncertainties in determining the imbalances of relative layer-transports in the deep layers are so great that the heavy weight given them in the Wunsch-Grant (1982) charts of absolute circulation yields results that are artificially dominated by bottom topography and are not realistic. We do not dispute the correctness of the formal inverse theory, only the results as given for the particular Atlantis section. We offer an elementary explanation for their peculiarity.

Given the magnitude of uncertainty in the relative imbalances it is suggested that they may possibly be insignificant, and that a simple two-point solution for the barotropic velocity can absorb most of them.

When, following Worthington (1976), a balance is sought for a subsection including only the Gulf Stream and the region immediately south of it, the uncertainties in the imbalances of layer transports are less. Application of inverse theory to a schematic representation of this subsection suggests that all possible solutions of the additive barotropic transport are too high to be readily acceptable and that, given his assumptions, Worthington's statement that a balanced recirculation across this subsection cannot be obtained is defensible.

1. Geostrophic imbalances on the 50W I.G.Y. Atlantis section

It is our intention to re-examine the evidence for a closed recirculation immediately south of the Gulf Stream, as revealed by inverse theory. The sample calculations are based upon a section made by Atlantis during the I.G.Y. from the Grand Banks to French Guiana, and displayed in the Fuglister Atlas (1960). Figure 1 shows this section divided up into four density-layers by isopycnals at $\sigma = 27.00$, $\sigma = 27.50$, and $\sigma_2 = 37.08$. Although the choice of isopycnal surfaces is arbitrary, these particular ones represent the main structural features of the hydrography. This choice corresponds roughly to the potential temperature layers chosen by Worthington (1976). The bottom is drawn from straight-line segments joining the observed depth at each station. The geostrophic transport for all station pairs of each of these layers relative to 2000 dbars (or to the bottom if the station pair is shallower than 2000 dbars) was computed. Transport functions of each layer and the

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Figure 1. Selected isopycnals on the Grand Banks to French Guiana section made by Atlantis on cruise 229.

sum for all four, obtained by cumulative summing from the northern (left in the figure) end, are displayed in Figure 2.

If the velocity at 2000 dbars truly vanished, if the section were representative of the long term mean, and if there were no flow across density surfaces in the oceanic region west of and bounded by the section, then we might expect that all of these transport functions, which begin at zero at the northern end of the section, would approach zero again at the southern end—at least within the bounds of departure from geostrophy that might arise from Ekman and other high-order dynamical effects.

It can be seen from Figure 2 that none of these transport functions firmly end at zero at the southern end. Moreover, due to their tendency to fluctuate from station pair to station pair—especially large at the southern end where the Coriolis parameter is small—we hesitate to assign values of these imbalances except within rather broad limits. It is plausible that these fluctuations on the scale of station-spacing represent transient features. Estimates of upper and lower bounds for these net geostrophic imbalances, as read from the graphs, are given in Table 1 under
the heading $T_t^*$. Within these bounds the imbalances of every layer might be zero—in which case the 2000 dbar reference level could be regarded as acceptable. On the other hand, by introducing a barotropic transport $b'$ [(Sverdrups per km depth)] at the indicated point (in band 39-41N) north of the Gulf Stream where the upper layer is thinnest, and a return transport $b''$ just south of the Stream where the upper layer is thickest (in band 35-37N), it is possible to reduce the imbalances to lie nearer to the center of the range. The thickness of the layers $h', h''$ corresponding to these two points are also given in Table 1. The equations for the balance are simply

Table 1. Form layer model—densities, layer thickness at recirculation points and bounds on relative transport imbalances estimated from Figure 2.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density range $\sigma_0, \sigma_2$</th>
<th>$h'$ (km)</th>
<th>$h''$ (km)</th>
<th>$T_t^*$ bounds ($10^6$Tons/sec) upper</th>
<th>lower</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\sigma_0 &lt; 27.00$</td>
<td>.193</td>
<td>.749</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$27.00 (\sigma_0) - 27.50 (\sigma_0)$</td>
<td>.236</td>
<td>.242</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$27.50 (\sigma_0) - 37.08 (\sigma_2)$</td>
<td>2.393</td>
<td>2.151</td>
<td>5</td>
<td>-2</td>
</tr>
<tr>
<td>4</td>
<td>$\sigma_2 &gt; 37.08$</td>
<td>2.478</td>
<td>2.158</td>
<td>25</td>
<td>-8</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>5.300</td>
<td>5.300</td>
<td>30</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 3. Graphical solution of “two-point” recirculation, on $b', b''$ plane, showing uncertainty of each constraint by width of bands. The solution is taken to be in the center of the region of the overlapping bands.

$$b'h'_i + b''h''_i + T_i^* = 0$$

for each layer, $i = 1, 4$. Because of uncertainty in the $T_i^*$, these equations define bands in the $b', b''$ plane, as shown in Figure 3. The common intersection of all these bands is roughly a narrow parallelogram whose maximum length extends from 0,0 to 10, -10. Thus it might be argued that the preferred values of $b', b''$ are 5, -5. When multiplied by the depth, 5.3 km, these figures represent an added barotropic transport of 27 Sverdrups eastward north of the Stream and a return flow of -27 (westward) south of the Stream. These two flows are shown as rectangles in Figure 4. For convenience of graphical display they are 200 km wide and 2.5 cm sec$^{-1}$ high.

Locating the position of the barotropic recirculation on the two sides of the Gulf Stream is fairly efficient in the sense that it utilizes maximum differences of layer thickness in the upper layer, $h_i$. The band in Figure 3 corresponding to $h_2$ is too broad to be a helpful constraint. Layer 3 is already nearly balanced. The transport of layer of thickness $h_4$ is very noisy. The location and amplitude of this 27 Sverdrup
recirculation is consistent with the currents of large amplitude at 4000 m at 55W reported by Schmitz (1978, 1980, this volume) but they do not correspond in detailed pattern, and there is uncertainty in being sure exactly how the directly observed currents are placed with respect to the instantaneous Stream, both of which wander somewhat in latitude with time.

It is, of course, possible to have a large recirculation somewhere on the section without affecting the geostrophic layer-budgets at all: the so-called null solutions, infinite in number. In fact, the pattern of three currents described by Schmitz at 55W might possibly straddle the Gulf Stream in such a way (a deep stream just beneath the offshore segment of the Gulf Stream, the two outlying counter-currents beneath the axis of the Stream and deepest part of the thermocline) as to constitute a null solution. To decide whether this is the case with precision in the presence of the known variability, may require a long-time monitoring of both currents and density structure near the Gulf Stream.

One can ask the question: is there any strong reason why one should choose to locate the recirculation in the region which this two-point model does? The not-
quite-satisfactory answer is that this is the region of strong slopes in the deep dy-
namic topography, that eddy-resolving numerical models generate the recirculation here, that direct current measurements by moored instrumentation show large am-
plitudes of mean velocity in the neighborhood, and that a region of strong maximum change in layer thickness is an efficient place to do budget balancing. A strong argument against this choice of location is the difference of water mass properties between Slope Water and North Atlantic Central Water.

It seems reasonable to adopt the conservative view that this rather arbitrary geo-
graphical placement of and crude determination of the amplitude of the recircula-
tion is about as far as we can go safely with budget-balancing with this single sec-
tion. However, it is tempting to venture further into more speculative and risky territory. Worthington (1976), Wunsch (1978) and Wunsch and Grant (1982) have travelled there before us. Their views invite discussion.

2. Budgeting using total values of imbalance over the complete section

In their inverse calculations of the general circulation of the entire North Atlan-
tic, Wunsch and Grant (1982, hereafter W-G) have included the 50W section, with ten density layers. With a starting reference level of 2000 dbars, W-G chose to present the eighth trial solution as their preferred one and have incorporated it into their overall oceanic scheme of circulation. For purposes of comparison we have summed their ten layers into our four. Table 2 shows the initial imbalance they used, and how these are reduced as the number of the trial solution (including more eigenfunctions) is increased. Figure 4 shows the computed reference level velocity \( b(\phi) \), for some of these trials where \( \phi \) is the latitude. The solutions are strongly similar to the bottom topography minus a constant: eastward flow over shallow seamounts and the coastal rises, westward currents over deeps. There is little similarity to the hydrographic structure in these solutions, and they actu-
ally transport more water than the two-point solution (although, of course, their \( \int b^2 H d\phi \) is smaller). Unless one can think of some compelling dynamical reason why there should be such dominant topographic control, one must surmise that this particular \( b \)-structure is a non-real artifact of the application of the inverse method to the section.

These solutions may be understood in fairly simple terms. Examination of the first W-G eigenfunction reveals that they have heavy weight on the rough bottom topography and on the bottom-layer (thickest). In the notation of Stommel and Veronis (1981) the solution of \( b(\phi) \) which minimizes \( \int b^2(\phi)H(\phi)d\phi \) is the form

\[
b = \lambda_0 + \sum_{i=1}^{n} \lambda_i \frac{h_i(\phi)}{H(\phi)},
\]

* This is the form of the minimizations used by Wunsch and Grant (1981) as well.
Table 2. Residual transport imbalances for Wunsch-Grant (1981) solutions for eleven trials summed into four layers.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density</th>
<th>Starting</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
<th>Trial 5</th>
<th>Trial 6</th>
<th>Trial 7</th>
<th>Trial 8</th>
<th>Trial 9</th>
<th>Trial 10</th>
<th>Trial 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt;27.00</td>
<td>.77</td>
<td>+1.43</td>
<td>-1.79</td>
<td>1.61</td>
<td>+1.65</td>
<td>9.70</td>
<td>7.58</td>
<td>7.54</td>
<td>8.74</td>
<td>1.87</td>
<td>1.51</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>27.00-27.50</td>
<td>-2.31</td>
<td>-8.74</td>
<td>-4.05</td>
<td>-3.74</td>
<td>-3.73</td>
<td>-3.18</td>
<td>-2.63</td>
<td>-2.65</td>
<td>-2.23</td>
<td>-.19</td>
<td>-.16</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>27.50-37.08</td>
<td>-7.75</td>
<td>-5.20</td>
<td>-0.43</td>
<td>-0.93</td>
<td>-1.02</td>
<td>-1.43</td>
<td>-4.35</td>
<td>-4.26</td>
<td>-4.98</td>
<td>-1.57</td>
<td>-1.20</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>37.08-45.90</td>
<td>8.65</td>
<td>11.16</td>
<td>2.65</td>
<td>2.95</td>
<td>3.00</td>
<td>1.87</td>
<td>.06</td>
<td>.02</td>
<td>-.80</td>
<td>-.20</td>
<td>-.10</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-7.61</td>
<td>-1.35</td>
<td>-.0431</td>
<td>-.109</td>
<td>-.107</td>
<td>-.041</td>
<td>.655</td>
<td>.645</td>
<td>.733</td>
<td>.093</td>
<td>.054</td>
<td>0</td>
</tr>
</tbody>
</table>
where $h_i(\phi)$ is the thickness of layer $i$, $H(\phi)$ is the total depth, $n$ is the number of layers upon which transport constraints are imposed and $\lambda_i$ are constants. If $H(\phi)$ is the strongest function of latitude ($\phi$), then it will dominate the solution for sizeable values of $n$. Qualitatively, on the 50W section, $H(\phi)$ is a more irregular function than most of the individual layers. A very simple interpretation can be given. Suppose, as in Figure 5 the bottom topography is simplified so that shallow portions are all 4 km deep, the rest of the ocean is 5 km deep. The bottom layer is 1 km thick. The function $b(\phi)$ must consist of two constant parts: one value over shallow, the other value over deep portions of the section. This is a very crude representation of the 50W section (it omits the form and location of the Gulf Stream for example). We now compute the transports associated with $b$. The $T_{1,*}$ Wunsch and Grant assign to the bottom layer is 6 Sverdrups. Therefore, for balance $b$ must provide for $-6$ Sverdrups, as shown. Since $b$ is independent of depth, this means an additional $-24$ Sverdrups has been added to the overlying...
4 km, thus we have introduced \(-30\) Sverdrups over the deep portion. But W-G's $T_i^*$ for the total transport over the section is \(-7\) Sverdrups, so that the total transport added over the whole section must be 7 Sverdrups. This means we must have 37 Sverdrups added over the shallow topography. These signs and magnitudes of $b$-transport correspond to the W-G solution [eastward flow over shallow regions, westward over the deep regions]. If we want velocity estimates, we must specify the fractions of total width of the section occupied by shallow water. It appears to be about 0.3. The section is 4000 km long, and hence the value of shallow water $b$ is \(+0.8\) cm s\(^{-1}\), which is comparable to that obtained over shallow topography in the W-G solutions in Figure 4.

One can recover the same transport results if the regions of $b$ are concentrated at certain portions of the section instead of being distributed uniformly over it. Minimization of the $\int b^2 H d\phi$ does reduce the velocities, but the transports are indifferent to it. This is true even in a more complicated and realistic case. A choice of form of the $b$-field restricted to a few point locations can actually reduce the barotropic transports below those determined by minimizing $\int b^2 H d\phi$.

In considering the relative certainty of various constraints on $T_i$ on a section across an ocean bounded to the west, it is tempting to consider the constraint on the total transport, $T_0 = 0$, as the most reliable physical statement of all: it is not affected by long-period storage as individual layers might be or mixing across isopycnals, the bottom topography $H(\phi)$ is not a function of time, and when applied by itself, leads to First Kind (see Stommel and Veronis, 1981) solutions which do not upset individual upper layer transports or heat transports much, do not introduce large amplitudes of computed $b$ anywhere (especially when the smoothed version produced by Wunsch's minimization of $\int b^2 H d\phi$ is used), and which is relatively well conditioned and associated with the largest eigenvalue in the problem.

Unfortunately, in practice, this apparent reliability is counterbalanced by the unreliability of the value of the total relative transport $T_0^*$. When calculating geostrophic transports relative to an intermediate depth reference level, the greater depths, larger station spacing, smaller vertical gradients, and ambiguity of how to calculate the dynamic topography in the presence of variable depth between station pairs, result in much larger uncertainties of geostrophic transport in deep layers than in layers within the main thermocline. We can expect therefore that errors in $T_0^*$ will be much larger than in the other $T_i^*$. Accordingly, practical use of the constraint $T_0 = 0$ (which otherwise seems so good) is of diminished significance.

Wunsch and Grant kindly reran their calculation of this section with diminished weights on the total transport, bottom two layers, and top layer. Trial solution 5 for this case is also shown at the top of Figure 4. This solution is more like the shape of the thermocline. It is spread more or less uniformly over the section, with westward flow mostly over the deepest portion of the thermocline at mid-latitudes, but is actually more complex than that. Of course we can again ask the question:
"Is there a compelling reason to suppose the sign of the barotropic flow to be correlated with depth (rather than slope) of the thermocline?"

Except for the eastward flow at low latitudes, this new Wunsch-Grant solution is rather similar to our two point one (the transports are spread over wider regions but in the same general location, and only slightly larger in amplitude). It might be argued that the large eastward flow at low latitudes of low salinity water between 400-1200 m is inconsistent with what we expect to find in Antarctic Intermediate Water, and the large changes in potential vorticity implied in the layer $27.00 \leq \sigma_0 \leq 27.50$ are problematical.

3. Budgeting for the limited-latitude case

Worthington (1976) has argued that the Gulf Stream/Recirculation system should not extend over the entire 50W section, but rather should be confined to the interior. His arguments are based partly upon the presence of very low salinity water at both the northern and southern ends of the section at depths shallower than 1000 m and the high salinity Mediterranean outflow. Although the regions of salinity anomaly do not necessarily act as a boundary to the flow if lateral mixing is allowed (Richardson and Mooney, 1975; Needler and Heath, 1975), there are other reasons for assuming the Gulf Stream/Recirculation is meridionally confined, for example the trajectories of floats.

Looking at the distribution with latitude of geostrophic transport in individual layers, shown in Figure 2, we see that the transport functions define a nearly closed cell in the slope water at the northern end of the section, and that the northern limit of the interior cell may begin close to the northern side of the Gulf Stream near 41N where the transports are close to zero. As we go south of 41N along 50W we find a sharp increase of eastward transport in the upper three layers (relative to 2000 dbars) with a westward transport in the deepest layer associated with the Gulf Stream, followed by a region of decreasing transport—the baroclinic recirculation, extending to 29N. At this latitude the lower two layers have nearly zero transport, while the upper two have leveled off to nearly constant values. This region of nearly constant transport in the upper layers and small variable transport in the lower layers extends from 29N to 20N at which latitude range we may define, for budget exercises, the southern end of the recirculation. The transport function and its variability here define the range of certainty of geostrophic imbalances. There is considerably less uncertainty in the interior imbalances and hence the bands on the $b',b''$ plane are much narrower than those for the full width of the section in section 1.

South of 17N, we encounter the salinity anomaly discussed by Worthington and the large changes in layer transports associated with the Guiana complex.

The limits of the total geostrophic layer transports between 41N and 29N are shown in Table 3. They are determined from values at nearby latitudes in Figure 2.
Table 3. Bounds for relative transport imbalances over limited latitude range [40°48′N-29°04′N].

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density</th>
<th>$T_i^*$ (10^6 Tons/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lower</td>
</tr>
<tr>
<td>1</td>
<td>&lt;27.00</td>
<td>+24</td>
</tr>
<tr>
<td>2</td>
<td>27.0-27.5</td>
<td>+8.5</td>
</tr>
<tr>
<td>3</td>
<td>27.5-37.08</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>37.08-bottom</td>
<td>-2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>38</td>
</tr>
</tbody>
</table>

As in section 1, the most efficient positions for a recirculation are those locations where the uppermost layer has the largest difference in thickness—on either side of the Gulf Stream. Using the thicknesses in Table 1 and the $T_i^*$ from Table 3, we construct the bands for the amplitudes of the recirculation $b', b''$ for each of the four layers, as shown in Figure 6. Because of lesser uncertainty in the $T_i^*$ the bands are quite narrow. We see from this calculation that it is not possible to define a simple recirculation which will reduce all individual layer transports to zero at a common $b', b''$ within the limits of uncertainty.

The essential reason for this lack of balance is that, over this region, both the top and second layers have positive transport imbalances, but opposite changes...
in thickness across the Stream. This occurs because the sum of their thickness is nearly constant south of the Stream—the surface $\sigma_0 = 27.50$ is nearly flat.

It does not seem that extending the recirculation over a wider region—separating the location of $b'$ and $b''$—can resolve this difficulty. However, we see from the equations (1, 2) in section 1 the slope of the bands in the $(b',b'')$ plane is determined by the ratio of the individual layer thicknesses. Moving $b''$ to lower latitude will increase the thickness of layer 2 at the expense of layer 1 (their sum is nearly constant over the region south of the Stream). The net effect will be to make the slope of the layer 2 band shallower while making the band for level 1 steeper. These bands now overlap in the vicinity of $b' \sim 80$, $b'' \sim 70$ (leading to an unrealistically large recirculation $0(400$ Sv) and still not intersecting the bands for the other layers). Introduction of more degrees of freedom in $b(\phi)$ (e.g., allowing it to be a continuous function) and seeking a minimal $b^2$ type solution in the interior leads to a very ill-conditioned solution of large amplitude, as we will see below when we invert the schematic example. Thus we essentially agree with Worthington’s (1976) statement that a budget of the transport by layers in the interior is not possible through an auspicious choice of reference level or recirculation cell. However, one need not conclude that geostrophy fails. The difficulty in getting a closed recirculation may be symptomatic of the need to include slope water and the Guiana Current in the real balance. In the subsequent section, we propose an alternative solution to this underlying dilemma.

4. A schematic model of the density field in mid-section

Except for details in the Slope Water north of the Gulf Stream and the more troublesome Guiana Complex at the southern end of the 50W section, the main structural features of the section can be described in terms of four layers divided by the three density surfaces $\sigma_0 = 27.00$, $27.50$ and a deep density surface $\sigma_2 = 37.08$.

The top layer appears to have nearly uniform potential vorticity from about $8N$ to the Gulf Stream, where its thickness becomes smaller and presumably the relative vorticity becomes appreciable. According to conventional ideas, this is the layer which is directly wind-driven and which should have the Sverdrup transport of the subtropical gyre. Schematically, in Figure 7, the depth of this layer is assumed to be simply proportional to latitude from $10N$ to the Gulf Stream. We rewrite the meridional distance $y$ from the equator in terms of $\nu$, normalized to the latitude of the Gulf Stream ($\nu = 1$). This places the southern limit of the slope at $\nu_1 = 0.2$. At $\nu = 1$ the underlying density surface reaches its maximum thickness $h_1$, and a short distance northward, across the Gulf Stream slopes abruptly up to the surface.

The density contrast from layer 1 to layer 2 is $\gamma_1$ (written as a fraction of the total density contrast from top layer to bottom layer of the ocean). The bottom of layer 2 is level between $\nu = .2$ to $\nu = 1.0$, at which its thickness is $h_2$, and then
Figure 7. A schematic representation of central density field, showing the parameters and transports of various portions of the schematic model.

it slopes upward parallel to the overlying surface. The normalized density contrast between layers 2 and 3 is $\gamma_2$. The bottom of the third layer is also level from $\nu = 0.2$ to a latitude $\nu_3$ (to be determined) at which it slopes downward toward the north, so that layer 3 also has uniform potential vorticity between $\nu = \nu_3$ and $\nu = 1$. The maximum thickness of layer 3 is $h_3$ at $\nu = 1$, whence its underlying density interface slopes upward parallel to the others and extends upward the same amount as those above. The contrast in density between the third and fourth layer is $\gamma_3$. The fourth, bottom layer rides upon the irregular bottom topography, but
we will assume that its velocity vanishes, so that the configuration of the ocean bottom is immaterial.

Subject to these structural constraints we will now explore the consequences of imposing constraints upon the transports of the three moving layers.

The transport of each of the three layers is

\[ T_1 = (\gamma_1 + \gamma_2 + \gamma_3) h_1^2/2 - (\gamma_1 h_1^2 + \gamma_3 h_1 h_a) (1 - \nu_3) - \gamma_1 h_1^2 (\nu_3 - \nu_1) \]

\[ T_2 = (\gamma_2 + \gamma_3) h_2 h_1 - \gamma_2 h_3 (h_1 + h_2) \ln (1/\nu_3) + \gamma_3 h_3 h_1 (1 - \nu_3) \]

\[ T_3 = \gamma_3 h_3 h_1 - (1 - \nu_3) \gamma_3 h_3^2 \]

The first terms on the right-hand side are the Gulf Stream transports; the second term is the transport south of the Gulf Stream, but north of \( \nu_3 \) (so that it lies in the "recirculation region"); the third term—which appears only in \( T_1 \)—is the wind-driven transport south of \( \nu_3 \).

Let us explore this model by choosing the structural features in order of importance.

**Case 1: One layer.**

First suppose \( \gamma_2 = \gamma_3 = 0 \), so that only the top layer is active \((T_2 = T_3 = 0)\). Then

\[ T_1 = \gamma_1 \left( \frac{h_1^2}{2} - (1 - \nu_1) h_1^2 \right) \]

Unless \( \nu_1 = 1/2 \), \( T_1 \) does not vanish. This is similar to the results of Fofonoff (1963) for a constant potential vorticity model. In general \( \nu_1 \approx 0.2 \) so we find a negative \( T_1 \). This occurs because the Coriolis parameter at low latitude is small, and hence more upper layer water flows westward than the Gulf Stream can carry away. The one layer geometry is similar to that used by Stommel et al. (1978), for a zonal section with uniform Coriolis parameter. In that case the Gulf Stream carries an excess of transport, in contrast with this same geometry along a meridional section. We can now see the role of the second layer—it permits a balance in the upper layer by increasing the Gulf Stream velocity there.

**Case 2: Two layers.**

Now assume that \( \gamma_2 \neq 0 \), but still we will take \( \gamma_3 = 0 \). Now it is possible to set the transports of both upper layers individually equal to zero

\[ T_1 = (\gamma_1 + \gamma_2) \frac{h_1^2}{2} - \gamma_1 h_1^2 (1 - \nu_1) = 0 \]

\[ T_2 = \gamma_2 h_2 h_1 = 0 \]

by the simple expedient \( h_2 = 0 \) and \( \gamma_2 = \gamma_1 (1 - 2\nu_1) \). The solution is not very realistic with \( h_2 = 0 \), but if \( h_2 \neq 0 \) then there will be an unbalanced transport in
the second layer under the Gulf Stream. In our opinion this is an important fact and demonstrates, even in this primitive schematic model, the difficulty of getting balance in the interior recirculation region.

It is, of course, possible that to the west of 50W there is a transformation of layer 1 water to layer 2 water of amount $-T_1$, affected by a heat loss in the Gulf Stream of $-T_1 \gamma_1 = \frac{H}{\theta^*}$, where $\theta^*$ is the temperature difference corresponding to the density difference from top to bottom of the ocean. In this case $T_1 = -T_2$. A graph showing the computed $h_1$, $h_2$, $\gamma_1$ and $\gamma_2$ for various heat fluxes $H$ across the section (positive eastward) is shown in Figure 8, under the assumption that the transport $S$ across the section south of the Gulf Stream is 35 Sverdrups, $\theta^* = 20^\circ C$, and the heat flux $H$ is in units of °C Sverdrups. The actual depth of the 27.00 and 27.50 density surface at 32°32'N and 50W (station 5444) and the values of $\gamma_1$ and $\gamma_2$ scaled from them are shown in Figure 9. Evidently $h_1 \approx 0.7$ km and $h_2 \approx 0.25$ km, which correspond to a heat flux $H$ of about $-80^\circ C$ Sverdrups. On the other hand, Figure 9 suggests that $\gamma_1 \approx 0.59$, $\gamma_2 \approx 0.41$ which corresponds to a heat flux $H$ (at the point labeled "b") of about $40^\circ C$ Sverdrups. The discrepancy cannot be easily circumvented in the two-layer case. It may be worth mentioning that according to Bunker’s (1982) maps of annual heat gain, the North Atlantic, west of 50W, does on the average lose heat at the rate of about 100°C Sverdrups, but much of this may be actually entirely by drop in temperature of waters within the top layer as it flows northward in the Gulf Stream without transformation to water in layer 2.
Case 3: Three layers.

We now introduce the third active layer, 3, anticipating that it may increase the transports in the recirculation region, and perhaps permit non-vanishing \( h_3 \) with zero heat transport. In Figure 5 a sample solution is shown, with \( \gamma_3 = 0.05, h_3 = 3.0, S = -0.35 \).

An immediate result from the condition that \( T_3 = 0 \) is that \( \nu_3 = 1 - \frac{h_1}{h_3} \), thus fixing the southern bound of the recirculation region (approximately \( \nu_3 = 0.7 \)) in fair agreement with observation.

Although the transport in the recirculation region is augmented, the value of heat-flux which corresponds to a realistic value of \( h_2 \) is not changed much from that of the two layer case. In fact the easiest way to reconcile the two-layer case (which has a net eastward \( T_2 \) and \( T_0 \) of approximately 6 Sverdrups, and a balanced \( T_1 = 0 \))
is to include the transports (relative to 2000 m) of the controversial Guiana complex \((T_1^* = 1, T_2^* = -9, T_0^* = -7)\).

This seems to mean that the main structural elements of the density field very nearly achieve geostrophic balance with relative velocity referred to the bottom layer [thus avoiding any influence of bottom topography on the velocity field] if one permits a modest conversion of layer 1 water to layer 2 water in the North Atlantic west of 50W, amounting to about 5 Sverdrups and presumably associated with the annual-mean rate of heat-loss of 100°C Sverdrups and perhaps with warm and cold eddy dissipation processes.

In summary, the schematic density model exhibits an imbalance of geostrophic layer-transport qualitatively similar to that first noted by Worthington (1976, p. 47) and also appearing above in the previous section. Because the thickness of layers at intermediate depth is so uniform in the vicinity of the Gulf Stream, it is difficult to construct \(b\)-fields of acceptable magnitude near the Stream which can reduce imbalances in the layers. We seem to be hoist upon the horns of the same trilemma. (1) We could abandon geostrophy—which would raise eyebrows. (2) We could extend the recirculation further to the south by adding a closed barotropic cell with eastward flow near the Gulf Stream and westward flow at low latitudes far to the south where layer 2 has a markedly different thickness from what it has near the Gulf Stream; but this is not very attractive because of the markedly differing salinity and potential vorticity in layer 2 near French Guiana. (3) We can assume that mass is not conserved within layers but that mixing due to rings and eddies at mid-depths, or a net heat transport involving a mass transformation of about 5 Sverdrups from layer 1 to layer 2 in the schematic model.

If pressed, we opt for a mixture of the latter two possibilities; it is not an attractive choice.

5. Inverting the schematic model

Let us assume that the schematic model shown in Figure 7, with the parenthetical figures, represents the true current system at 50W in the segments excluding slope water (lat > 41N) and the Guiana current (lat < 17N). Although the currents approach the bottom in the recirculation regions (segments 2 and 3) they actually vanish everywhere at the bottom. Thus, by hypothesis, the bottom topography does not actually appear explicitly in the budgets. Now suppose that we have a slightly imperfect version of the three layer thicknesses in Figure 7, that there are small apparent imbalances \(T_i^*\) and that we attempt an inverse calculation.

Let us begin by assuming that width of the Gulf Stream vanishes insofar as its contributions to the integrals of the equations for the Langrange multipliers. The minimal solution (Stommel and Veronis, 1981) is \(b = \lambda_1 h_1 + \lambda_2 h_2 + \lambda_3 h_3\), and the multipliers are determined by the equations
\[ \lambda_1 \int_{0.2}^{1} h_1^2 dy + \lambda_2 \int_{0.2}^{1} h_2 dy + \lambda_3 \int_{0.2}^{1} h_3 dy = -T_1^* \]
\[ \lambda_1 \int h_1 h_2 dy + \lambda_2 \int h_2^2 dy + \lambda_3 \int h_3 dy = -T_2^* \]
\[ \lambda_1 \int h_1 h_3 dy + \lambda_2 \int h_2 h_3 dy + \lambda_3 \int h_3^2 dy = -T_3^* \]

or

\[0.2573 \lambda_1 + 0.1746 \lambda_2 + 1.0001 \lambda_3 = -T_1^*\]
\[0.1746 \lambda_1 + 0.2259 \lambda_2 + 0.8607 \lambda_3 = -T_2^*\]
\[1.0001 \lambda_1 + 0.8607 \lambda_2 + 4.2142 \lambda_3 = -T_3^*\]

The determinant of the left-hand side is very small

\[ \Delta = 0.0005082 \]

and consequently the system is very ill-conditioned. The results are so sensitive to small errors in the observed quantities that dropping the fourth decimal place in the elements of the matrix reduces the determinant to 0.000294. Within 0.1% error in the elements the determinant is not significantly different from zero. It is impossible to make a meaningful inversion of this system under these circumstances. Because the determinant hovers about zero, the minimum value of \( \int b^2 d\phi \) will be very large. This is an instance of the power of the inverse method: it assures us in this case that even the minimum solution of all possible solutions is uncomfortably large.

If we should decide to risk the perils of including a representation of the bottom layer in the inversion of the schematic model, the slightly unbalanced system would fall under control mainly of the bottom topography.

6. Conclusion

Calculating absolute geostrophic velocity on the Atlantis 50W I.G.Y. section leads to awkward results. The large fluctuations in cumulative relative transports of individual layers toward the southern end of the section introduce uncertainty in estimates of the long-term imbalance in them.

The latitudinal dependence of the particular solutions offered by Wunsch and Grant (1982) using the minimization of \( \int b^2 H d\phi \) (or \( \int b^2 d\phi \)) over the whole section depends heavily on the bottom topographical details such as the isolated Corner Rise Seamounts, and the continental rise. This is partly due to the weighting employed, heaviest—on the thickest layers, and partly because the thicknesses of the upper density layers are not strongly linearly-independent, and hence the numerical solution is forced to depend on details in bottom topography and, to a lesser extent, small scale—presumably transient—irregularities in layer thickness on this 50W section. Writers on inverse theory know all this in principle, but in
presenting their results have not yet found a way to emphasize how hypothetical and tentative they really are. We do not dispute the correctness of the formal inverse theory, nor are we inappreciative of its great interest and potential.

In an effort to gain more insight into the pattern of flow across 50W we have constructed some elementary solutions: also hypothetical and tentative. The two-point solution spanning the Gulf Stream can absorb the layer imbalances over the entire section within conservative estimates of their uncertainty, but ignores what may be important differences in water-mass properties. An attempt to obtain balance with a two-point solution within Worthington’s recirculation portion of the section leads to the same dilemma that he discovered. The schematic representation of three upper layers in the limited latitude portion of the section (used to suppress from the outset the influences of bottom topography, small scale eddies and poorly known deep-layer transport) illustrates what main features of the density field lead to the imbalance, and suggests that a relatively small downward flow of about 5 Sverdrups across the isopycnal interface at \( \sigma_z = 27.00 \) in the ocean west of 50W could restore the balance. An indication of the unlikelihood of finding any exact steady-state, no mixing, closed geostrophic balance within the limited latitude schematic model is obtained by formally applying the inverse theory (in the sense of Stommel and Veronis) to the schematic model itself. It shows that of all the infinite number of possibilities the minimal solution itself has an extraordinarily large value of \( \int b^2dy \) and therefore all other possible solutions have an even greater \( \int b^2dy \). This may be an example of the power of the inverse method: setting lower bounds, and finding them unacceptably high.

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