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A comparison of geostrophic and measured velocities in the Equatorial Undercurrent

by S. P. Hayes

ABSTRACT

Profiles of zonal velocity in the upper 500 m at the equator are computed from the differentiated form of the geostrophic relation \( u = -\frac{1}{\beta \rho} \frac{\partial \rho}{\partial y} \) using four CTD sections at 110W and one section at 125W. The meridional curvature of the pressure field between 1N and 1S was estimated by fitting a second order polynomial to the dynamic height (relative to 500 db) at each pressure surface. Computed profiles all indicated an equatorial undercurrent at an approximate depth of 70 db and amplitude of 150 cm s\(^{-1}\) relative to the minimum speed near 200 db. The rms deviation between computed and directly measured velocity differences from the undercurrent core to 250 m was 34 cm s\(^{-1}\) which is about 20% of the mean difference. These results provide strong evidence for the importance of geostrophy within 1° of the equator.

1. Introduction

Observational and theoretical studies have suggested that the Equatorial Undercurrent (EUC) may be in geostrophic balance. Early measurements (e.g., Knauss, 1960; Montgomery and Stroup, 1962) pointed out that the meridional structure of dynamic height at the undercurrent core (referenced to some deep level) was consistent with geostrophy. On the other hand, Knauss (1966) presented meridional density sections which did not indicate geostrophic balance. Further observations (e.g., Taft and Jones, 1973; Taft et al., 1974) found qualitative evidence of a geostrophic component to the EUC although temporal variability was considered too large to permit quantitative comparisons. A geostrophic EUC of 150 cm s\(^{-1}\) requires only a 2.2 dyn cm height difference between 0° and ± 1°. This small difference is difficult to measure because of high-frequency variability in the density field.

Theoretical models of the EUC often assume geostrophic balance in the meridional momentum equation (Moore and Philander, 1977; McCreary, 1981). This balance also holds for the low-frequency variability if this variability can be described by linear equatorial trapped waves (Moore and Philander, 1977). Recent

1. Pacific Marine Environmental Laboratory/NOAA, 3711 15th Ave., NE, Seattle, Washington, 98105, U.S.A.
measurements (Eriksen, 1981; Ripa and Hayes, 1981) suggest that time variability at the equator may be dominated by internal Kelvin modes. At low frequencies these modes are geostrophic. Long zonal wavelength Rossby modes which may also contribute low-frequency fluctuations in the zonal current at the equator are also nearly geostrophic. Eriksen (1982) had reasonable success in computing profiles of deep zonal currents near the equator using geostrophy.

At the equator the geostrophic relation for the zonal current (using conventional notation):

$$\beta y u = - \frac{1}{\rho} \frac{\partial p}{\partial y}$$

becomes indeterminant if the meridional pressure gradient vanishes. However, several authors (e.g., Jerlov, 1953; Arthur, 1960) have noted that the derivative of this equation provides a means of estimating the zonal velocity at $y = 0$:

$$\beta u = - \frac{1}{\rho} \frac{\partial^2 p}{\partial y^2}$$

Arthur (1960) discussed the approximations involved in this equation and concluded that it offered the best hope of computing $u$ if one could obtain reasonably accurate estimates of the meridional curvature of the pressure field. However, because of this difficulty most studies have used Eq. (1) and attempted to estimate $u$ to within ± 0.5° of latitude. Montgomery and Stroup (1962) noted that with only a few hydrographic stations (e.g., 0°, ± 1°) Eqs. (1) and (2) were computationally equivalent. In the present study we use CTD data collected at approximately 25 km intervals from 1N to 1S to estimate the curvature of the dynamic height as a function of depth. Computed zonal velocity profiles are compared with currents measured either by a velocity profiler or moored current meters. The general agreement between computed and measured currents offers strong evidence for the importance of geostrophy at the equator.

2. Observations

In recent years we have made several CTD sections across the equator at 110W and one at 125W as part of the NOAA Equatorial Pacific Ocean Climate Studies (EPOCS). In order to reduce the effects of high-frequency and small-scale density fluctuations on the computation of zonal velocity, we concentrated our analysis on four sections which had nine meridionally distributed stations between 1N and 1S. These sections were taken at 110W on 5 and 13 February and 6 May 1979 and at 125W on 13 July 1977. An additional 110W section (6 to 9 March 1981) with seven CTD stations and four sections with coarser station spacing were also considered for comparison. CTD processing and calibration techniques used on these data are discussed in Mangum et al. (1980). All calculations were based on profiles
which had been vertically averaged to obtain values at 1 db increments. Directly measured velocity data were available simultaneously for February 1979 and March 1981 sections. In July 1977 (May 1979) there was a delay of three (five) days between current measurements and the sections. Velocity data were provided by moored arrays (July 1977; February and May 1979), by a cable tethered Düing profiling current meter (PCM) (February 1979), and by a free-fall acoustically tracked velocimeter (TOPS) (March 1981). The moored data were all provided by Halpern (personal communication, 1980) and the 110W measurements are discussed in Halpern (1980a, 1980b). The PCM data were provided by Firing (personal communication, 1980). TOPS and the associated data processing techniques are briefly discussed in Hayes (1981). For the equatorial profile presented here, TOPS was acoustically tracked using a system similar to the White Horse profiler (Luyten et al., 1981). The near-equatorial TOPS profiles were computed from relative velocity measurements (Hayes, 1981) referenced at 500 db to the absolute velocity profile at the equator.

Density sections from 3N to 3S at 110W and 125W (Fig. 1) all showed the equatorial spreading of the pycnocline which suggests qualitatively that the EUC could be in geostrophic balance. The dynamic height (relative to 500 dbar) for a pressure surface near the EUC core (80 dbar) had a local maximum near the equator (Fig. 2). The locations of the dynamic height minima varied between transects, but were confined equatorward of ± 2.5°. In general, the dynamic height was not symmetrical about the equator although the local maximum height always occurred within 25 km of the equator. The shape of this curve is determined by the total flow within which the EUC is imbedded. As noted by Leetmaa (1982) strong westward near-surface flow is often observed just north of the equator and eastward flow sometimes extends to at least 2S. The complexity of the equatorial current profiles is seen in TOPS profiles from 1N to 1S (Fig. 3). During this section surface flow at the equator and to the south was eastward while north of the equator it was westward. The apparent core of the EUC rose from about 100 db at 30'N to about 50 db at 1S. The extra-equatorial currents can lead to an asymmetrical dynamic height profile with considerable height difference between 1N to 1S. However, as observed, the meridional slope at the equator can still be negligible.

The dynamic height measurements are also subject to considerable variability associated with high-frequency, small meridional scale fluctuations. As discussed in Gregg (1979) and Eriksen (1982) relative differences in dynamic height due to random errors in the CTD measurements are negligible. Differences associated with bias (i.e., calibration) errors between stations may be larger. However, since all stations on a given section were collected with the same CTD and since (except for the March 1981 section) all stations were collected within a 24 h period it is thought that calibration changes are not significant. Real high-frequency, small-scale density fluctuations probably dominate. One measure of these fluctuations
(which also includes calibration-related variability) comes from CTD time series at a single location. Time series near 110W were collected at 0° in February 1979 (24 casts over 48 h) and at 0° and ± .65° in May 1979 (12 casts over 13 h at each location). Also, at 125W a 13 h time series was made on the equator. These series suggest that the high-frequency variability leads to an uncertainty in the dynamic height near the EUC core (relative to 500 dbar) of ± 1.1 dyn cm in a 24 h period. This uncertainty may be time dependent if the high-frequency internal wave field near the equator is nonstationary (Hayes, 1981).

3. Geostrophic calculation

On each section, at specified pressure surfaces, the dynamic height relative to 500 db from 1N to 1S was fit to a second order polynomial \( D(y) = a_0 + a_1 y + a_2 y^2 \)
in the meridional coordinate, \( y \), using a least squares procedure. Each curve fit involved nine data points and, hence, six degrees of freedom. Such fits were made at 10 dbar increments from 0 to 200 db and at 50 db increments from 200 to 400 db. The latitude limits of the sections were chosen to be \( \pm 1^\circ \) in order to reduce the effects of the extra-equatorial currents; the polynomial regression included a first order term in order to allow for height changes from 1N to 1S which these currents can cause. Zonal velocity at the equator was estimated using Eq. (2):

\[
\frac{u}{\beta} \frac{\partial^2 D}{\partial y^2} = - \frac{20}{\beta} a_2
\]

where \( \beta = 2.3 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \) is the gradient of the Coriolis parameter at the equator. Error estimates for \( u \) are provided by the standard error of \( a_2 \) obtained from the least squares regression.

Figure 4 shows geostrophic and measured velocity profiles for the four densely sampled sections. Moored currents shown are daily averaged values. In each case the deepest moored current measurement was used as a reference level velocity and the computed (and PCM) velocities were adjusted to agree with this level. This adjustment accounts for the possibility of a nonzero meridional curvature in the 500 db pressure field which could be associated with the deep equatorial jets which have been observed in this region (Hayes and Milburn, 1980; Leetmaa and Spain, 1981). The maximum 500 db velocity inferred from this curvature is 45 cm s\(^{-1}\) in May 1979.
Agreement in shear from 50 db to 250 db is quite good for all the 110W profiles. The depth and velocity at the core of the undercurrent are well represented by the geostrophic profile. Above the EUC in the near-surface layer, agreement becomes poor and error bars on the velocity estimates are large. As shown by the temperature profile, the thermocline begins at about 25 db. Above this, wind forcing is
Figure 4. Geostrophic and measured velocity profiles at 110W and 125W. The geostrophic ($U_g$, solid) and PCM ($U_p$, dashed) velocities were referenced to the deepest moored current measurement. Moored currents (+) are daily averaged values. In February measured and computed currents were simultaneous, in May the moored data are from 1 May and in July from 10 July. The dotted curve ($T$) shows the equatorial temperature profile.
probably more important and one does not expect the geostrophic approximation to be valid. Below the upper layer, best agreement was obtained with the February sections which had simultaneous velocity data. The root mean square (rms) difference between measured and computed velocity for the 10 estimates from 50 m to 200 m was 23 cm s$^{-1}$ for these two sections. Considering the large shear in this region and the error estimates on the calculated velocity, this agreement is excellent.

Similar results hold for the 125W data. In this case the moored current meters do not resolve the EUC core depth and these measurements ended three days before the section was taken. Periods when the maximum velocity was at either 50 or 100 m are observed in the moored time series (Halpern, personal communication, 1980). The geostrophic profile indicates a velocity maximum of 210 cm/s at 60 db. Again, agreement with measurements deeper than this core is good. Above the core the water is more weakly stratified and poorer agreement between measured and computed currents at 50 m was found.

A final comparison of velocity profiles was provided by the section in March 1981. Since only seven CTD casts from 1N to 1S over a four-day period were available, the error limits on the computed velocities were generally higher. However, reasonable agreement with the directly measured velocity profiles (Fig. 5) from
TOPS was again obtained. As noted above, the meridional sampling with TOPS showed large variability in the equatorial band. This spacial structure may explain some of the discrepancies seen in the profiles in Figures 4 and 5. Since the geostrophic velocity estimate must represent a spacial average, it may be more appropriate to compare the computed velocity with the mean velocity from 30°N to 30°S. It is clear from Figure 5 that, in this case, the geostrophic profile is more representative of such a meridional average.

Success with these densely sampled sections encouraged us to examine four additional sections at 110W (3 June and 23 October 1979, 13 August and 10 September 1980) where station spacing was about 50 km (five stations from 1N to 1S); hence, the second order regression had only 2 degrees of freedom. Baroclinic velocity estimates (referenced to 500 db) near the EUC core (80 db) were $58 \pm 13 \text{ cm s}^{-1}$ (June 1979), $-88 \pm 104 \text{ cm s}^{-1}$ (October 1979), $1 \pm 39 \text{ cm s}^{-1}$ (August 1980) and $123 \pm 103 \text{ cm s}^{-1}$ (September 1980). These results are much more variable than those seen in Figures 4 and 5. In fact, the October section indicated westward flow at 80 db although the error bars are large. This result depended strongly on the single station at 1N which had anomalously warm water throughout the thermocline (Mangum et al., 1980) and a dynamic height (80 db/500 db) 4 dyn cm greater than .5N or 1.5N. Removing the 1N station from the regression but keeping the number of degrees of freedom constant by extending the fit to 1.5N yielded an estimated eastward velocity of $57 \pm 51 \text{ cm s}^{-1}$. Results for the other sections were also quite dependent on single stations. The variability found in these more crudely sampled sections was larger (compared to the 25 km sampled sections) than expected based on high-frequency variability alone. A Monte-Carlo simulation assuming a rms dynamic height uncertainty of 1 dyn cm and a symmetric EUC with a velocity maximum of 150 cm s$^{-1}$ showed only a 20% increase in the uncertainty of velocities computed with 5 rather than 9 points. However, since the observed dynamic height curve is not symmetric, the estimated curvature may be more sensitive to the number of stations in the section.

4. Conclusions

Geostrophic velocity profiles at the equator were computed from five meridional CTD sections from 1N to 1S. For the four sections at 110W the average directly measured velocity difference from 75 m to 250 m was $158 \pm 8 \text{ cm s}^{-1}$ (uncertainty quoted is the standard error of the mean); the average geostrophic velocity difference weighted according to the uncertainty in each estimate was $136 \pm 25 \text{ cm s}^{-1}$. These mean values agree to within one standard error. The rms deviation between measured and computed velocity difference was $34 \text{ cm s}^{-1}$ or about 20% of the measured velocity difference. In all cases the deviation was less than one standard error of the geostrophic current estimate based on the least squares polynomial
regression. Thus, within the limits of the experiment, measured currents were in geostrophic balance. Expressed as a relative error, the discrepancy between computed and measured currents at the equator (20%) is quite similar to that found at mid-latitudes (26%) (Bryden, 1977). However, the absolute deviations are much larger at the equator (34 cm s\(^{-1}\)) than at mid-latitude (1.9 cm s\(^{-1}\)). Because of the large uncertainty in the computed velocity, these data cannot determine the extent to which variations in the EUC are in geostrophic balance. Measurement limitations are principally due to high-frequency fluctuations which from 70 db to 500 db have a dynamic height standard deviation of about 1 dyn cm in a day. Uncertainty in the estimated current could be reduced by repeated densely sampled CTD sections. Such an experiment should include velocity profiling in order to define the meridional structure of the near-equatorial currents.

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