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Variability in the central equatorial Indian Ocean, Part II: Oceanic heat and turbulent energy balances

by Michael J. McPhaden

ABSTRACT

Heat and turbulent energy balances in the central equatorial Indian Ocean are examined using over two years (January 1973-May 1975) of simultaneous oceanic and atmospheric measurements from the island of Gan (00°41'S, 73°10'E). Results of a mixed layer analysis indicate that about 80% of the observed variance in mixed layer temperature, or equivalently, sea-surface temperature (SST) on monthly time scales can be accounted for by observed surface heat flux. Horizontal advection is of secondary importance because even though zonal currents are strong in this region, horizontal temperature gradients are very weak. Entrainment mixing is generally negligible, though on occasion enough turbulence is generated by wind stirring to penetrate the thermocline. A major source of uncertainty in these calculations, equivalent to about 20% of the observed SST variance, is the parameterization of surface fluxes from bulk formulae. Results of a heat budget analysis for the upper 200 m indicate that heat content over this depth range is primarily controlled by vertical displacements of the thermocline associated with time varying geostrophic currents rather than by surface fluxes.

The presence of an equatorial SST minimum in the Atlantic and Pacific Oceans and its absence in the Indian Ocean is rationalized in terms of wind-driven circulation patterns and mixing processes. In the Atlantic and Pacific Oceans, easterlies generate cool surface temperatures via equatorial upwelling, a process whereby cold water in the thermocline is brought close to the surface and mixed in the high shear zone above the Undercurrent. In the Indian Ocean, westerlies suppress upwelling by depressing the thermocline and driving eastward currents with relatively little vertical shear.

1. Introduction

Recent studies have indicated that the climate system is particularly sensitive to low-latitude changes in sea-surface temperature (SST). Prescribed change experiments (e.g., Shukla, 1975; Rowntree, 1976) have shown that equatorial SST anomalies have a larger effect in the overlying atmosphere than comparable mid-latitude anomalies. Moreover, these effects can be global in character as Bjerknes (e.g., 1966, 1969) suggested in his early work on meridional teleconnections. A growing number of observational studies (e.g., Namias, 1978) add credence to these model results and further draw attention to the critical significance of tropical SST in the global climate system. It is clear, therefore, that effective climate modeling
and prediction will require a better understanding of the factors that control tropical SST than we have at present.

SST is also of interest to tropical oceanographers because it is often used as an indicator of subsurface processes. For example, there is a correlation between dynamically controlled changes in thermocline depth and SST in the eastern Pacific during El Nino (McCreary, 1976) and in coastal regions within the Gulf of Guinea during upwelling season (Moore et al., 1978). The physical basis for this correspondence is that when cold thermocline water is carried closer to (further from) the surface, it is easier (harder) to mix across the base of the mixed layer. This in turn leads to colder (warmer) SST. Such correspondence is not universal however because other processes, e.g., surface heat flux and horizontal advection, can also affect SST. To quantify the connection between SST and subsurface processes, it is therefore necessary to explicitly determine the relative significance of various terms in the surface heat balance.

Variations in SST may also induce important dynamical effects in the tropical oceans. McPhaden (1981) has shown in a steady state model that the response to zonally varying SST patterns is two orders of magnitude larger in the equatorial oceans than at mid latitudes and is comparable in both magnitude and spatial structure to the wind-driven equatorial circulation. This strong dynamical effect is due to the sensitivity of geostrophic currents to small changes in horizontal thermal gradients at low latitudes. Proper representation of this coupling between thermodynamics and dynamics requires accurate parameterization of small scale mixing processes in the surface boundary layer. Yet these parameterizations are only crudely specified in the present generation of tropical ocean circulation models.

The purpose of this study is to examine SST variability in the central equatorial Indian Ocean on monthly time scales from an analysis of the heat and turbulent energy balances in the mixed layer. Section 2 describes the data, which consist of nearly two years of simultaneous oceanic and atmospheric measurements from the island of Gan (00°41’S, 73°10’E), and the derivation of surface heat flux estimates. In Section 3, various terms in the mixed layer heat budget are estimated and it is found that most of the variability in mixed layer temperature (which is synonymous with SST since there are no vertical temperature gradients in the mixed layer) is due to variability in surface heat flux. It is also shown that horizontal advection is of secondary importance and that entrainment, as determined from relative changes in mixed layer and thermocline depths, is active on only a few occasions. Variations in the heat content of the upper 200 m are calculated and found to be controlled primarily by vertical motion of the thermocline associated with time varying geostrophic currents rather than by surface fluxes. In Section 4, the time history of entrainment velocity is reproduced via the turbulent energy equation and wind stirring is identified as the most important mixing mechanism. Section 5 is a summary of major results and a discussion of equatorial upwelling.
Figure 1. (a) Zonal winds and zonal currents averaged over 20 m between 0-20 m, 60-80 m, and 160-180 m for the period Jan 1973-May 1975. Smooth line is low passed version consisting of monthly means. Stippling indicates the presence of an undercurrent. (b) Same as (a) except for meridional winds and currents. (c) Same as (a) except for temperature and mixed layer depth. (d) Same as (a) except for depths of the mixed layer, 26°C, 20°C, and 14°C isotherms.

2. The data

Knox (1976) and McPhaden (1982, hereafter referred to as Part I) have extensively discussed wind, ocean current and temperature data from the island of Gan (00°41'S, 73°10'E) for the period January 1973-May 1975. Time series of these quantities from Part I have been reproduced in Figure 1. Mixed layer depth (MLD) in Figure 1c, 1d is defined as the first depth below the surface at which the vertical temperature gradient exceeds 5°C/100 m. In this study these data are augmented by three hourly observations of air temperature ($T_a$), dew point temperature ($T_d$)
and cloud amount (C) at the same location from the NCAR archives. Though they are island measurements, both $T_d$ and $C$ are representative of open ocean conditions (Ramage et al., 1972) and are therefore suitable for use in estimating surface oceanic heat fluxes. Air temperature recorded on the island may not be as representative (Ramage et al., 1972) though this is of little consequence for heat budget studies since sensible heat, which depends on air-sea temperature contrasts, is typically only a few Wm$^{-2}$ in the tropical oceans.

Surface heat flux is estimated by

$$ Q_o = Q_R + Q_S + Q_L $$

where $Q_R (= Q_{sw} - Q_{lw})$ is the net radiative flux at the sea surface, $Q_{sw}$ is incoming shortwave radiation, $Q_{lw}$ is outgoing longwave radiation, $Q_S$ is sensible heat flux and $Q_L$ is latent heat flux. These have been evaluated from the bulk formulae

$$ Q_{sw} = Q_{cs}(1 - \beta)(1 - 0.65C) $$

$$ Q_{lw} = eBT_o^4(0.39 - 0.056q_s^{1/2})(1 - 0.53C) + 4eBT_o^5(T_o - T_a) $$

$$ Q_S = \rho_a C_B C_p U(T_a - T_o) $$

$$ Q_L = C_B L U(q_a - q_o) $$

Wind speed, saturation humidity, absolute humidity, and specific humidity are given by $U$, $q_o$, $q_a$ and $q_s$, respectively. Sea-surface temperature ($T_o$) is approximated by a 20 m depth average. Clear sky radiation ($Q_{cs}$) was obtained from Budyko (1956). Emissivity ($\epsilon = 0.98$), Boltzmann’s constant ($B = 5.67 \times 10^{-6}$Wm$^{-2}$K$^{-4}$), ocean albedo ($\beta = 0.06$), latent heat of evaporation ($L = 2440$ Jg$^{-1}$), heat capacity at constant pressure ($C_p = 3.94$ Jg$^{-1}$C$^{-1}$), air density ($\rho_a = 1.15 \times 10^{-3}$g cm$^{-3}$) and density of sea water ($\rho = 1$ g cm$^{-3}$) are all constant. The exchange coefficient ($C_E$) was set to $1.2 \times 10^{-3}$ (Large and Pond, 1982).

Weekly and monthly estimates of $Q$ are shown in Figure 2. Monthly data were derived by smoothing weekly estimates with a 61 day running mean filter and resampling every 30.5 days. (See discussion of Fig. 5 for the choice of this filter length.) Net shortwave minus longwave radiation ($Q_R$) is plotted since $Q_{lw}$ is fairly constant (=60 Wm$^{-2}$) over the record length. Variations in $Q_R$ are therefore mostly due to variations in $Q_{sw}$. Net radiation shows a strong 1 cycle per year (cpy) variation even though at these latitudes a 2 cpy variation dominates clear sky radiation. This is due to the fact that cloud cover is higher during the southwest monsoon than during the northeast monsoon thereby eliminating the expected September extremum in $Q_{cs}$. Latent heat flux exhibits a strong 2 cpy fluctuation that is clearly related to the winds. Sensible heat flux shows a similar variation though this component is in general at least an order of magnitude weaker than other flux terms. Total surface flux is typically into the ocean. It is highest in boreal winter when the...
winds are light and the skies are relatively clear; it is lowest during the transition periods of April/May, October/November when the winds are intense and the sky is overcast.

Means of $Q_R$, $Q_L$ and $Q_S$ are 106, -72, and 6 Wm$^{-2}$, respectively, where negative values denote fluxes out of the ocean. The mean of $Q_o$, the total heat flux, is 28 Wm$^{-2}$ into the ocean. These estimates agree surprisingly well with those calculated from 60 years of ship data near Gan (Hastenrath and Lamb, 1980) and suggest that statistics derived from this 2-1/2 year data set are representative of those from other periods. Conversely, in Part I a comparison of wind stress statistics for the period 1973-1974 with the previous 10 years yielded the opposite conclusion. In view of this conflicting evidence, results of the following 2 sections should be generalized to other periods only with great caution.

3. The heat balance

a. Mixed layer. The depth integrated heat balance for the mixed layer, neglecting lateral turbulent diffusion, is

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T + \frac{1}{h} w \omega \Delta T = \frac{1}{\rho C_p h} [Q_o - Q_h - I_o e^{-\gamma h}] . \quad (3)$$
The base of the mixed layer is located at \( z = -h \) where \( Q_{-h} \) is vertical turbulent heat diffusion and \( \Delta T \) is a temperature jump due to entrainment mixing. Entrainment velocity, \( w_e \), is defined only when the layer deepens relative to any variations in thermocline depth. Horizontal vector velocity is designated \( u \). The variable \( I_o \) represents penetrative radiation which is \( \approx 45\% \) of incident solar flux (Ivanov, 1977); \( \gamma \) determines its attenuation with depth.

To examine the heat balance on monthly time scales, subtract the mean from equation (3) to get

\[
\frac{\partial T}{\partial t} - \frac{1}{\delta_t} [T(t_1) - T(t_0)] + (u \cdot \nabla T) + \left( \frac{1}{h} w_e \Delta T \right)' = \frac{1}{\rho C_p} \left[ \left( \frac{Q_o}{h} \right)' - \left( \frac{Q_{-h}}{h} \right)' - \left( \frac{I_o}{h} e^{-\gamma h} \right)' \right]
\]

where primes denote fluctuations about the mean, e.g., \( (Q_o/h)' = Q_o/h - (Q_o/h) \), etc. and \( \tau = t_1 - t_0 \) is record length. The term preceded by \( 1/\delta_t \) corrects for trends introduced by the fact that the end points of the time series are at different temperatures. Time series of observed \( T, h, u \) and \( Q_o \) can now be used to estimate the relative importance of various terms in (4). For the turbulent diffusive flux term, this requires a model in which \( Q_{-h} = \rho C_p K \frac{\partial T}{\partial z} \) where \( K \) is a vertical eddy diffusivity and \( \partial T/\partial z \) is the temperature gradient below the base of the mixed layer. Calculation of the entrainment term requires estimates of both \( w_e \) and \( \Delta T \) which are described below.

The simplest balance to test with the present data set is one in which horizontal advection, penetrative radiation, entrainment and turbulent fluxes are neglected. Then integration of (4) yields

\[
T_o(t) = \frac{1}{\rho C_p} \int_{t_0}^{t} \left( \frac{Q_o}{h} \right)' dt + \frac{1}{\delta_t} [T(t_1) - T(t_0)] (t - t_2) + T(t_2)
\]

where \( T_o \), an estimated mixed layer temperature, is derived from a single heat source, viz. the surface flux \( Q_o \). The integration constant \( T(t_2) \), where \( t_0 < t_2 < t_1 \), is arbitrary and chosen such that the average \( T_o \) over the record length is equal to the average of the observed mixed layer temperature. Thus (5) predicts fluctuations and not absolute values of mixed layer temperature.

Figure 3 shows a calculation of \( T_o \) based on a trapezoidal rule integration of observed \( (Q_o/h)' \). Superimposed on this is the observed mixed layer temperature, \( T_{obs} \). Though there are some 0(1°C) discrepancies between these two curves, most notably in April 1973, the rms temperature difference \( \sigma_{\Delta T} = [(T_{obs} - T_o)^2]^{1/2} \) where overbar denotes time average, is only 0.32°C. Moreover, the coherence squared between these curves is 0.81, i.e., 81% of the variance in mixed layer temperature.
can be accounted for by considering only surface fluxes. This is a remarkable result, given the simplicity of (5).

One might have expected zonal advection to dominate the heat balance at Gan since the zonal currents are so strong in this region (Fig. 1a). It is therefore important to check the consistency of the above result by independently determining the magnitude of advection as well as other physical processes. Likewise, it is important to examine the sources of error that can affect the comparison of $T_q$ and $T_{obs}$, viz. errors due to the numerical integration of $(Q_o/h)'$, random observation errors in $T_{obs}$, $h$ and $Q_o$ and bias errors due to imperfect parameterizations of the surface fluxes. Table 1 summarizes the results of a scale analysis in which the 0.32°C discrepancy between $T_q$ and $T_{obs}$ is rationalized in terms of both these errors and the physical processes neglected in (5). The first entry in each category is an expected rms temperature amplitude on monthly time scales, $\langle \sigma_{Tq} \rangle$, which is probably accurate to within a factor of 2. The second entry is a variance ratio, $R = \langle \sigma_{Tq} \rangle^2 / \sigma_{AT}^2$, which indicates the fraction of the discrepancy between $T_q$ and $T_{obs}$ that can be explained by each category assuming all categories are independent of one another. What follows is a detailed derivation of these results.

Table 1. Expected rms amplitude, $\langle \sigma_{AT} \rangle$, in °C on monthly time scales due to numerical and observational errors, inaccurate parameterizations of surface fluxes and various physical processes. $R = \langle \sigma_{AT} \rangle^2 / \sigma_{AT}^2$ where $\sigma_{AT}$ is the observed rms discrepancy between $T_q$ and $T_{obs}$. See text for discussion.

<table>
<thead>
<tr>
<th>Numerical</th>
<th>Observational</th>
<th>Parameterization</th>
<th>Physical processes</th>
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</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$T_{obs}$</td>
<td>$h$</td>
<td>$Q_o$</td>
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<tr>
<td>$\langle \sigma_{AT} \rangle$</td>
<td>0.05</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>$R$</td>
<td>0.02</td>
<td>0.10</td>
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</table>
The expected error due to numerical integration is based on standard formulae (e.g., Carnahan et al., 1969) and at 0.05°C is clearly negligible compared to \( \sigma_{\Delta T} \). From the discussion of errors in Part I, uncertainties in \( T_{\text{OBS}} \) and \( h \) on monthly time scales are < 0.1°C and < 10 m, respectively. The latter translates into a \( \sigma_{\Delta T} \) of at most 0.08°C based on the scale argument

\[
<\sigma_{\Delta T}> \approx \frac{1}{\rho C_p} \frac{\tilde{Q}_0}{h} \left( \frac{\sigma_h}{h} \right) \Delta t
\]

with \( \sigma_h = 10 \text{ m}, \tilde{Q}_0 = 30 \text{ Wm}^{-2}, h = 50 \text{ m} \) and \( \Delta t = 1 \text{ month} \). It therefore is unlikely that any of these sources contribute significantly to the observed \( \sigma_{\Delta T} \).

It is difficult to estimate the uncertainty in \( \tilde{Q}_o \) due to random observational errors since so many variables are involved. Therefore this source of error is neglected and, for the sake of argument, it is assumed that only systematic errors due to imperfect parameterizations are important. Based on a 10% uncertainty on the parameterizations of radiative (Simpson and Paulson, 1979) and turbulent (Large and Pond, 1982) fluxes, one can estimate an expected bias error of \( \approx 20 \text{ Wm}^{-2} \). This translates into an uncertainty, derived from

\[
<\sigma_{\Delta T}> \approx \frac{1}{\rho C_p \Delta t} \sigma_q \Delta t
\]

of \( \approx 0.26°C \) which is clearly comparable to \( \sigma_{\Delta T} \). It is moreover a lower bound on the total expected uncertainty due to \( \tilde{Q}_o \) because of the neglected random component of \( \sigma_q \).

The magnitude of various physical processes can also be estimated using arguments similar to (6) and (7). For instance, one can show that zonal advection is strong enough to account for all the discrepancy between \( T_q \) and \( T_{\text{OBS}} \) based on \( \sigma_u \approx 30 \text{ cm sec}^{-1} \) on monthly time scales (Fig. 1a) and \( \partial T/\partial x < 0.5°C \text{ per 10° longitude (Colburn, 1974)} \). On the other hand, meridional advection is likely to be one to two orders of magnitude smaller since \( \sigma_v \ll \sigma_u \) and \( \partial T/\partial y < \partial T/\partial x \). The probability that turbulent diffusion contributes significantly to observed temperature variability is small based on an assumed eddy coefficient of \( K = 1 \text{ cm}^2/\text{sec} \) and observed monthly variations in \( \partial T/\partial z \) below the mixed layer. Likewise, penetrative radiation can account for only a small percentage of the observed \( \sigma_{\Delta T} \) if one assumes \( I_o = 0.45 \text{ Q}_{sw} \) and \( \gamma = (12.5 \text{ m})^{-1} \).

To evaluate the effects of neglected turbulent entrainment in (5) requires estimates of both \( w_e \) and \( \Delta T \). The first of these is given by \( w_e = w > 0 \) where

\[
w = \frac{\partial h}{\partial t} + w_{-h}.
\]

In this expression, \( \partial h/\partial t \) is the time rate of change of mixed layer thickness and \( w_{-h} \) is a vertical velocity in the thermocline just below the base of the mixed layer. The
inequality means that entrainment occurs only when the layer deepens relative to changes in thermocline depth which, if \( Q_{-h} \) is assumed negligible, can be approximated by

\[
w_{-h} = \frac{\partial z}{\partial t} \bigg|_{z = -h_{26}}
\]

where \( h_{26} \) is the depth of the 26°C isotherm. This isotherm is chosen because it is always in the thermocline just below the mixed layer (Fig. 1d). Deeper isotherms will not give representative values of \( w_{-h} \) because vertical coherence scales in the thermocline are only tens of meters (Part I). The calculation of \( w \) from finite difference equivalents of \( \partial h / \partial t \) and \( \partial h_{26} / \partial t \) is shown in Figure 4. While entrainment will be discussed in more detail in Section 4, it is sufficient for now to note that significant entrainment velocities of \( 0(10^{-4} \text{ cm sec}^{-1}) \) occur only during the transition months of 1973. Detrainment, or periods when \( w < 0 \), will also be discussed later.

The temperature jump, \( \Delta T \), at the base of the mixed layer due to entrainment is not always clearly defined in observed temperature profiles, but it can be approximated as

\[
\Delta T = \Delta h \frac{\partial T}{\partial z} = w_e \Delta t \frac{\partial T}{\partial z}
\]  

(10)

where \( \Delta h \) is the change in mixed layer depth due to entrainment over 1 month and \( \partial T / \partial z \) is a 20 m average temperature gradient below the mixed layer.

According to this expression, magnitudes of \( \Delta T \) and \( \Delta h \) for the entrainment events evident in Figure 4 are \( \approx 1^\circ \text{C} \) and \( \approx 10 \text{ m} \) for \( \partial T / \partial z \approx 10^{-3}^\circ \text{C} \text{ cm}^{-1} \). This amount of mixing would result in a drop of \( 0(0.1^\circ \text{C}) \) over a 50 m mixed layer in one month or an equivalent downward heat flux of \( \approx 10 \text{ Wm}^{-2} \). The timing of these
Figure 5. Mean square temperature difference between $T_{\text{obs}}$ and $T_q$ as a function of low pass filter length in days. Error bars indicate the 95% confidence limits for the true value of a $\chi^2$ distributed random variable.

Entrainment episodes is such that they would reduce, but not eliminate, the discrepancies between $T_q$ and $T_{\text{obs}}$ in the spring and fall of 1973. For the record as a whole however, entrainment is generally not active and therefore can account for only about 6% of the variance between $T_q$ and $T_{\text{obs}}$.

In addition to the foregoing scale analyses, the sensitivity of $T_q$ to variations in mixed layer depth was tested by integrating (5) with $h$ held fixed at its mean value of 55 m derived in Part I. Results indicated little difference between this and the variable depth integration because rms variability in $h$ on monthly time scales is significantly less than the mean mixed layer depth. The sensitivity of $T_q$ to variations in filter length was also checked by integrating (5) for filter lengths ranging from 15.25 days to 152.5 days. As can be seen in Figure 5, filter lengths shorter
than 61 days fail to remove uncorrelated high-frequency variability. Conversely, longer filter lengths suffer from decreased reliability due to a reduction in the degrees of freedom. Hence, one can conclude that for this data set, the 61 day filter is optimal.

b. Upper 200 m. It is instructive to determine the effect that variations in surface flux have on the heat content of the upper 200 m vis a vis the mixed layer. For convenience (5) is used in its differential form

$$\rho C_p h \left\{ \frac{\partial T}{\partial t} - \frac{1}{\delta_t} [T(t) - T(t_0)] \right\} = Q_o'$$

where $T$ is a 200 m average temperature, $h (= 200 \text{ m})$ is constant and $Q_o' = Q_o - \overline{Q_o}$. The left side of (1) is defined as $H'$ and compared to $Q_o'$ in Figure 6. It is clear that fluctuations in the observed depth averaged heat content are typically an order of magnitude larger than those predicted by $Q_o'$. This is not due to error sources or physical processes like those listed in Table 1, but to the fact that the thermocline moves vertically to balance time varying geostrophic currents (Part I). This result agrees with those of other analyses (e.g., Oort and Vonder Haar, 1976; Schopf, 1980; Hastenrath and Lamb, 1980; Merle, 1980) which have shown that when the heat balance is referenced to a fixed depth within or below the tropical thermocline, surface fluxes are of secondary importance compared to internal dynamical effects on seasonal and longer time scales. The important point to make here is that vertical displacement of the thermocline may have little or no influence on SST variability and to a large extent can be filtered out by referencing the heat balance to the mixed layer depth as was done in (5).
4. Turbulent energy equation

In the previous section it was shown that, in general, entrainment was not important on monthly time scales. An analysis of the turbulent energy equation, in which the time history of $w_e$ is reproduced, confirms this result. In addition, this leads to a determination of the relative importance of various mixing mechanisms in the surface boundary layer.

Niiler and Kraus (1977) have discussed at length appropriate forms of the depth integrated turbulent energy equation for the mixed layer under a wide variety of oceanographic circumstances. For horizontally homogeneous turbulence in steady state equilibrium, a reasonably general expression is

$$\frac{1}{2} g \alpha h \Delta T [1 - s Ri^{-1}] w_e = m u_*^3 - \frac{1}{2} \frac{g \alpha h}{\rho C_p} F(Q_o)$$

where $g = 9.8 \times 10^2 \text{ cm sec}^{-2}$ is gravity, $\alpha = 2.5 \times 10^{-4} \text{C}^{-1}$ is the fractional change in density due to temperature, $Ri = g \alpha (\partial T/\partial z)/| \partial u/\partial z |^2$ is a gradient Richardson number, $u_* = | \tau/\rho_o |^{1/2}$ is a friction velocity and

$$F(Q_o) = \begin{cases} Q_o, & Q_o > 0 \\ nQ_o, & Q_o < 0 \end{cases}$$

The sources of turbulence generation in (12) are shear instability at the base of the mixed layer (parameterized by $Ri$), wind stirring ($u_*^3$) and free convection due to surface cooling ($Q_o < 0$). Sinks of turbulent energy in (12) are due to entrainment of denser thermocline water into the mixed layer (proportional to $w_e \Delta T$), surface heating ($Q_o > 0$) and dissipation. This latter is parameterized in terms of empirically determined efficiency factors $m$, $n$, and $s$ for the generating mechanisms. The coefficient $s$ can also be interpreted as a critical Richardson number above which shear production is effectively dissipated.

In order to calculate $w_e$ from (12), it is necessary to assign values to $m$, $n$, and $s$. Acceptable values of $n$ and $s$ range between (0,1). Zero implies that all turbulence generated by shear instability or free convection is dissipated before any mixing occurs. Conversely, unity implies none is dissipated. The only limitation on $m$ is that it be non-negative, i.e., wind stirring cannot be a sink of turbulent energy. To set values exactly, one can use the results of laboratory experiments (e.g., Kato and Phillips, 1969), though such results do not universally apply to all oceanic situations (Niiler and Kraus, 1977). Another strategy and the one adopted for this paper, is to determine the efficiency factors from the oceanic data itself. This requires that there be distinctly different regimes when one generating mechanism dominates the others so each efficiency factor can be independently determined (e.g., Davis et al., 1981a, b).

Figure 7 is a plot of monthly averaged quantities that contribute to turbulence
Figure 7. Sources of turbulent energy generation: free convection (proportional to $Q_o > 0$), wind stirring (proportional to $u^3$) and shear instability (proportional to $Ri^{-1}$). Dashed line indicates $s^{-1} = 1$. Shear instability is an effective source of turbulence for entrainment only when $Ri^{-1} \geq s^{-1}$ (i.e., $Ri \leq s$).

generation. In each case, monthly means were obtained by first calculating weekly estimates and then smoothing with a 61 day running mean filter. Surface flux (which has already appeared in Figure 2, but is reproduced here for completeness) is such that the ocean is significantly cooled only during spring 1973. For the most part, therefore, $Q_o$ is acting as a sink of turbulent energy. Wind stirring shows a strong 2 cpy variation characteristic of the zonal winds (Fig. 1a). Note that the most intense entrainment event in Figure 4 coincides with the strongest wind stirring episode in fall 1973. Also shown in Figure 7 is the inverse Richardson number $Ri^{-1}$ calculated from velocity and temperature changes over a 20 m depth range immediately below the mixed layer. (This quantity is plotted rather than $Ri$ since the latter is nearly singular at times.) Note that at mid-latitudes, the earth’s rotation limits the effectiveness of shear-induced mixing to shallow mixed layers on time scales of a pendulum day (e.g., Pollard, et al., 1973). No such constraints are in force near the equator so that one can assume shear production is proportional to a slowly varying Richardson number. Figure 7 shows, however, that assuming no dissipation of shear generated turbulence, i.e., $s = 1$, this mechanism should seldom cause entrainment since $Ri^{-1} \geq s^{-1} = 1$ in only 3 of 29 months. Moreover, if there is any dissipation whatsoever, $s < 1$ so that the dashed line moves to higher values and shear generation is even less effective. This result is not an artifact of the 20 m vertical average since 10 m averages yield a similar time history of $Ri^{-1}$. Thus, as
in the case of $Q_o$, it is unlikely that shear instability is a primary source of turbulent energy.

The above discussion justifies an initial null hypothesis of $n = s = 0$. To determine $m$, one then integrates (12) over those months for which the ocean is being heated. If it is assumed that $w_e \approx 0$ based on discussion in Section 3, one gets

$$0 = mu_s^3 - \frac{1}{2} \frac{g\alpha}{pC_p} hQ_o$$

(14)

which is equivalent to the statement that the observed average mixed layer depth should be the average Monin-Obukhov depth calculated from $Q_o$ and $u_s^3$. The value of $m$ required for this balance to hold is 0.38. Note that $u_s^3$ and therefore $m$ calculated from (14) are nonlinear functions of $\tau$ and depend on the averaging interval over which stress is computed. Thus $m$ would be lower (perhaps by a factor of 2) had $u_s^3$ been calculated from 3 hourly rather than weekly estimates of wind stress. This is of no consequence for the present study since $m$ and $u_s^3$ always appear in combination in (12). However, it is useful to bear in mind this dependence on averaging interval when comparing values of $m$ derived by different investigators.

One can now use (12), with $\Delta T$ given by (10), on a month by month basis to calculate $w_e$. Results are shown in Figure 4 as a bar graph superimposed on estimates of $w_e$ derived in the previous section. Only values for which entrainment deepening exceeds an assumed noise level of $10^{-4}$ cm sec$^{-1}$ are plotted. There is good agreement between the two different representations of $w_e$ in both magnitude and timing of entrainment events. Specifically, both indicate that virtually all significant entrainment proceeds at a rate of $0(10^{-4}$ cm sec$^{-1}$) and is confined to the spring and fall transition months of 1973. Moreover, the calculation of $w_e$ from (12) is relatively insensitive to the magnitudes of $n$ and $s$. The only effect of varying $n$ from 0 to 1 is to approximately double the May 1973 value of $w_e$. Likewise for $s$ between 0 and 1, in general, neither the magnitudes nor patterns of entrainment change significantly. Thus, the entrainment velocity $w_e$ is well modeled by (12) with wind stirring as the primary source of turbulence generation.

Analysis of the turbulent energy balance can also provide insight into the detrainment events alluded to in Section 3. These occur in the winter of both 1973 and 1974 when winds are light and surface heating is intense. Under such conditions, the mixed layer will anomalously shoal until a new equilibrium Monin-Obukhov depth is established. The intensity and duration of these events are also comparable to the entrainment events described above, consistent with the fact that there are no long term trends toward mixed layer deepening or shoaling over the 29 month record.

5. Summary and discussion

Simultaneous oceanic and atmospheric measurements from January 1973 to May
1975 were used to examine the heat and turbulence energy budgets near Gan. Results indicate that surface heat flux accounts for about 80% of the observed mixed layer temperature, or equivalently SST, on monthly time scales and that zonal advection is of secondary importance. Entrainment mixing is negligible most of the time, though on occasions sufficient turbulence is generated by wind stirring during the transition months to penetrate the thermocline. A major source of uncertainty in these calculations is the parameterization of surface heat flux from bulk formulae which introduces errors equivalent to ≈20% of the observed monthly SST signal.

Changes in the heat content of the upper 200 m were estimated to illustrate the effect that choice of reference level has on the heat balance. In this example, changes in heat content were primarily controlled by vertical displacements of the thermocline associated with time varying geostrophic currents. By contrast, surface fluxes were an order of magnitude smaller.

Mean monthly charts of SST in the Indian Ocean (Hastenrath and Lamb, 1979) do not show evidence for an equatorial minimum like that found over most of the Atlantic and Pacific. In these latter two oceans, prevailing easterlies produce Ekman divergence near the equator which is balanced by an upward motion in the thermocline. This brings cold water close to the surface where it is mixed by turbulence generated in the high shear zone between the SEC and Undercurrent. The net result of this coupled dynamic/thermodynamic process known as equatorial upwelling, is that SST at the equator is lower than either north or south. The circulation patterns and mixing processes in the Indian Ocean are quite different, however. An undercurrent may develop during the latter part of the Northeast Monsoon, but it is a short-lived, transient phenomena (Fig. 1a and Part I). It is more typical for winds to blow from the west which favors Ekman convergence and hence depression of the thermocline. Zonal currents are also generally from the west with little shear across the base of the mixed layer. Thus, in the Indian Ocean, equatorial upwelling is suppressed and an SST minimum does not develop.

Despite these major differences between oceans, the atmospheric and oceanic conditions in the Pacific west of the dateline in many respects resemble those in the central Indian Ocean. Both regions are characterized by monsoon winds, weak horizontal SST gradients, deep mixed layers and the absence of an SST minimum. Hence it may be that in the western Pacific, SST variations on monthly time scales are determined primarily by surface fluxes as well.

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