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The potential flow over ripples on the seabed

by A. G. Davies

ABSTRACT

This paper describes an analytical method to determine the potential flow over an undulating surface of prescribed shape. The method is applied both to profiles of real sand ripples on the seabed and to idealized ripple profiles. The flow is assumed to be oscillatory, uniplanar, deep and nonseparating, and the ripples to be symmetrical about their crests.

The method involves the perturbation of a uniform oscillation by the introduction of a repeated pattern of discrete singularities, such that one of the streamlines takes the required shape of the ripple. The solution then enables the prediction of the velocity field close to the rippled surface, in relation to the unperturbed free stream flow.

The velocities at the crest and trough positions on the bed surface are shown to be strongly dependent both upon the ripple steepness (height to length ratio) and upon the degree to which ripple crests are peaked in relation to their flatter troughs. The heights above the bed to which ripples have an influence on the flow, are shown to be about $0.6L$ and $0.8L$ at the crest and trough positions respectively, where $L$ is the ripple wavelength.

1. Introduction

The nature of the flow over undulating topography has been the subject of a variety of theoretical and experimental investigations. However, little of this work is of direct use in explaining the flow over the irregular features which are found on natural beds of sand in the sea. The motivation for the study described in this paper was the need to interpret measurements of the wave induced velocity field obtained above the crests of naturally occurring sand ripples, in an investigation the aim of which was to compare sediment threshold motion conditions on rippled and flat beds (Davies and Wilkinson, 1978). The bed features in the former case were fossil ripples, not in equilibrium with the prevailing flow conditions and not undergoing any overall change of shape with time. No flow separation occurred from the bed surface and the occasional to- and fro- motion of sediment was confined to the region of the ripple crests. In the latter case, sediment movement occurred over the entire bed surface.

One of the main conclusions from this study was a large apparent discrepancy between values of the velocity amplitude at the threshold of motion in the two cases.

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In particular, for ripples of height 12cm and length 85cm, the critical free stream velocity amplitude (measured in the unperturbed free stream flow at a height of 100cm above the bed) was smaller than the value for a flat bed composed of the same material by a factor of about 2. The same effect has been noted in the laboratory studies of Carstens, Neilson and Altinbilek (1969). The present paper concentrates on the prediction of the flow over general ripple shapes with a view to explaining discrepancies of this type quantitatively.

The model described is of the potential flow over symmetrical ripples of finite amplitude, with emphasis being placed both on the importance of ripple height to length ratio (or steepness) and on the ripple shape, that is the extent to which the ripple crests are peaked compared with their flatter troughs. The importance of the solution described is that it enables the calculation of the velocity field over any prescribed ripple shape, as well as such quantities as the thickness of the zone of influence of ripples above the bed surface. This information is essential in the interpretation of experimental results, enabling the flow remote from the bed to be related to the flow at the bed surface in the assumed absence of frictional effects. Thus, apparent discrepancies in such parameters as sediment threshold velocities over rippled and flat beds can be understood.

More generally, the relevance of a potential model of the type described is that it provides an exact picture of the frictionless flow over topography to compare with solutions involving friction. Thus frictionless effects due to topography can be separated from viscous effects, throughout the flow field. In addition, at least in the case of a laminar boundary layer, it should be possible to make use of results obtained with the proposed method to assess local variations in the skin friction over a ripple profile, by scaling the equivalent shear stress on a flat bed by the ratio of the potential velocity at the ripple surface to the unperturbed free stream velocity.

2. The model of flow over a rippled bed

Theory. The model is of a deep nonseparating irrotational flow above a rippled surface. It is assumed that the ripples have a given profile which is repeated indefinitely along the axis of flow, and that they are symmetrical about their crests. The flow over the ripples is simulated by superimposing irrotational flow solutions; in particular, a uniform oscillation in the x-direction is perturbed by introducing, close to the axis of flow, a repeated pattern of discrete singularities, such that one of the streamlines of the resulting motion is distorted into the desired ripple shape. In the proposed method, the singularities have been taken as doublets with periodic strengths of amplitude determined by the ripple feature being modelled, each doublet introduced permitting a pair of coordinate values to be specified on the ripple surface. The singularities being all on one side of the streamline defining the ripple surface in the \((x, y)\) plane, the potential on the other side gives the unique solution of the problem. The solution can be readily computed for any required ripple shape.
Since the ripples which are formed by an oscillatory flow are symmetrical, the analysis is presented for such a flow. However, if a pattern of symmetrical ripples is unmodified by some new flow, which may be steady or unsteady, the results obtained in this paper will still be valid since time appears only as a parameter. Furthermore, the oscillatory flow over a fossil bed of this type may have an orbital diameter which is much greater or much less than the wavelength of the ripples.

The flow to be perturbed is defined by the complex potential

$$w = U_z \cos \sigma t$$

(1)

where \(w = \phi + i\psi\), \(z = x + iy\), \(U\) is the velocity amplitude, \(\sigma\) is the frequency of the oscillation and \(t\) is the time. The velocity components \((u, v)\) in the directions \((x, y)\) respectively are defined by

$$\frac{dw}{dz} = -u + iv \quad \text{or} \quad u = -\phi_x = -\psi_y \quad v = -\phi_y = \psi_x$$

(2)

\(\phi\) being the velocity potential and \(\psi\) the stream function. Since the ripple pattern is to be repeated indefinitely in the direction of flow, the complex potential of an infinite row of doublets, each of strength \(\mu\) and positioned at \(z = 0, \pm L, \pm 2L \ldots \), is used:

$$w = \frac{\mu \pi}{L} \cot \left( \frac{\pi z}{L} \right)$$

(3)

This result holds for doublets with axes lying on the x-axis and can be derived by an argument analogous to that given by Milne-Thomson (1968, § 13.71).

Suppose initially that the complex potentials (1) and (3) are added, subject to the additional assumption that the doublet strength \(\mu\) is periodic and given by \(\mu = \mu_0 \cos \sigma t\). Then

$$w = \left\{ U_z + \frac{\mu_0 \pi}{L} \cot \left( \frac{\pi z}{L} \right) \right\} \cos \sigma t$$

(4)

and this corresponds to the stream function

$$\psi = \left[ U_y - \frac{\mu_0 \pi}{L} \sinh \left( \frac{2\pi y}{L} \right) \right] / \left\{ \cosh \left( \frac{2\pi y}{L} \right) - \cos \left( \frac{2\pi x}{L} \right) \right\} \cos \sigma t$$

(5)

If the zero streamline is now considered, it can be seen that \(\psi = 0\) for all \(t\) where

$$U_y \left\{ \cosh \left( \frac{2\pi y}{L} \right) - \cos \left( \frac{2\pi x}{L} \right) \right\} = \frac{\mu_0 \pi}{L} \sinh \left( \frac{2\pi y}{L} \right)$$

(6)

Clearly this equation is satisfied by all points on the x-axis apart from the doublet positions. However there are two further solutions of (6), namely zero streamlines which reflect one another about the x-axis and are symmetrical about the y-axis;
that is, if \((x_0, y_0)\) satisfies (6), so also do \((- x_0, y_0), (x_0, - y_0)\) and \((- x_0, - y_0)\). It can be shown that, if \(\mu_0\) is sufficiently large, these two streamlines do not meet at any point; in particular, if \(\mu_0 > U \frac{L^2}{\pi^2}\) no positions will occur where a zero streamline divides.

The two zero streamlines occupy the same positions in the \(z\)-plane for all \(t\), and are the only streamlines for which this is true. Furthermore, they display a spatial periodicity with wavelength \(L\) in the \(x\)-direction, each doublet deflecting the streamlines away from it. In what follows, the zero streamline in \(y > 0\) is identified with the fixed ripple surface, and the flow is examined in the upper half plane on the side of this streamline where no singularities occur.

The velocity components \((u, v)\) corresponding to (4) are

\[
\begin{align*}
  u &= \left[ -1 + 2\pi^2 \left( \frac{b}{L} \right)^2 \frac{\{1 - \cosh 2\pi Y \cos 2\pi X\}}{\{\cosh 2\pi Y - \cos 2\pi X\}^2} \right] U \cos \sigma t \\
  v &= \left[ 2\pi^2 \left( \frac{b}{L} \right)^2 \frac{\sinh 2\pi Y \sin 2\pi X}{\{\cosh 2\pi Y - \cos 2\pi X\}^2} \right] U \cos \sigma t
\end{align*}
\]

where \(X = \frac{x}{L}\), \(Y = \frac{y}{L}\) and \(b^2 = \frac{\mu_0}{U}\). It can be seen that at ripple crests and troughs, at \(X = 0\) and \(X = \frac{1}{2}\) respectively, the vertical velocity is zero. Also, for large \(Y\), \(u \rightarrow -U \cos \sigma t\) and thus an unperturbed flow exists well above the rippled bed.

The simple formulation outlined above permits the zero streamline in \(y > 0\) only one degree of freedom, namely through the value specified for \((b/L)\). However, more general ripple shapes can be generated by the introduction of additional rows of doublets of type (3) into the solution. Thus, for instance, a row of doublets placed at the 'half points', \(\pm \frac{L}{2}, \pm \frac{3L}{2}, \ldots \) etc., can be introduced, each having the same strength, but this strength differing from \(\mu\). This permits two points to be specified in each ripple length, say at the crest and trough positions, and, by introducing further rows of doublets, more ripple coordinates can be specified in each ripple wavelength. The number of specified coordinates must equal the number of rows of doublets introduced, and ultimately the strengths of the doublets in each row are uniquely determined by the particular coordinate values chosen. In principle, this procedure can be extended indefinitely by the inclusion of more and more doublets, until finally a continuous distribution is attained.

In the numerical calculations described later, ripple shapes have been specified by either eight or sixteen points per ripple length, with the crest at \(X = 0\) and the trough at \(X = \frac{1}{2}\). Since the ripples have been assumed to be symmetrical about their crests, doublets which are equidistant from a crest have been given the same strength. It has been found also, in the interests of computational efficiency and ac-
accuracy of solution, that it is advantageous to displace the doublets off the $X$-axis. In the general solution, the $Y$ coordinates of the doublet positions are expressed $Y_j$, $j = 1, N$, from crest to trough, where there are $2(N-1)$ doublets per ripple length and due to symmetry $Y_j = Y_{2N-j}$, $j = 2, N-1$. These values $Y_j$ have been determined from the rule

$$Y_j = \bar{Y}_j \quad \text{ripple surface} - Y_d, \ j = 1, N$$

in which the displacement $Y_d$ has been assigned a fixed numerical value for any one set of calculations.

Writing $w = W(z) \cos \sigma t$ and $Z = \frac{z}{L}$ the general solution for the amplitude of the complex potential has been taken as

$$W(z) = ULZ + \frac{\mu_1 \pi}{L} \cot \pi (Z - iY_j) + \frac{\mu_{N \pi}}{L} \cot \pi (Z - \frac{1}{2} - iY_N)$$

$$+ \sum_{j=2}^{N-1} \frac{\mu_j \pi}{L} \left[ \cot \pi \left\{ Z - \frac{(j-1)}{2(N-1)} - iY_j \right\} + \cot \pi \left\{ Z - \left( 1 - \frac{(j-1)}{2(N-1)} \right) - iY_j \right\} \right]$$

in which $\mu_j$, $j = 1, N$, are the amplitudes of the doublet strengths. Equation (10) is the solution for the case of $2(N-1)$ doublets per ripple wavelength, and for which the ripple pattern is symmetrical about its crests and troughs (typically $X = 0$ and $\frac{1}{2}$ respectively). The corresponding stream function and velocity components have been obtained from Eq (2).

**Numerical Calculations.** The procedure in the numerical calculations involved, firstly, specifying the ripple shape by $N$ coordinate points per half ripple wavelength $(X_j, \bar{Y}_j; j = 1, N)$ and choosing a value for $Y_d$ in Eq (9), according to the criterion given below. Since the zero streamline passes through each point specified, the equation $\text{Im} W(z) = 0$ is satisfied $N$ times, enabling the doublet strengths $\mu_j$, $j = 1, N$, in (10) to be obtained. The $N$ linear simultaneous equations for the $\mu_j$ were solved by a matrix inversion procedure, and the value of $Y_d$ was chosen to be as large as possible, subject to the restriction that the matrix inversion could be accomplished. Where a particular combination of the specified ripple coordinates and $Y_d$ led to an ill-conditioned matrix, a new smaller value of $Y_d$ was selected to resolve the difficulty. In practice, this criterion ensures the smoothest curve through the prescribed points, and optimum values of $Y_d$ have been found to lie in the range $(0.07 L, 0.16 L)$ for the various ripple profiles examined. The determination of the doublet strengths $\mu_j$ in (10) gives the unique solution for the complex potential at the general point $(X, Y)$, corresponding to a particular set of specified values $(X_j, \bar{Y}_j)$.

The position of the zero streamline between the specified points was determined from Eq (10), and each simulated ripple shape was checked since on certain, but
unusual, occasions sign differences occurring among the doublet strengths $\mu_j$ made the existence of a closed zero streamline possible around one or more doublets in each ripple length. Solutions of this kind were rejected and the difficulty overcome by the choice of a new value for $Y_d$. Acceptable solutions were normally such that the $\mu_j$ were all positive, and by extension of the argument following Eq (6), the zero streamline was found to be nondividing and everywhere forced away from the row of singularities to pass smoothly through the specified points as required.

Ripple shape factor. It has been found convenient to classify ripples both in terms of their steepness and of their shape, that is the extent to which the ripple crests are peaked in comparison with their flatter troughs. Ripple steepness has been defined in the usual way as $(H/L)$ (see Fig. 1(i)) where $H = \overline{Y_1} - \overline{Y_N}$ and a "shape factor" $S$ has been defined as follows:

$$S = \frac{4l}{L}$$  \hspace{1cm} (11)
where $l$ is the horizontal distance from the crest to the mid-height position on the ripple. For a purely sinusoidal ripple $S = 1$, while for a ripple with a more peaked crest $S < 1$. Although the parameter $S$ appears to be a relatively crude guide to shape, it is shown later to be important in determining velocity variations close to the ripple surface.

3. Evaluation of the method

Results for the typical case of a sinusoidal bed. To illustrate the general character of the solutions obtained, the case of a purely sinusoidal bed with $H/L = 0.1$ ($S = 1$) has been taken. Firstly, in Figure 1(i) the positions of certain streamlines at time $t = 0$ are shown; the zero streamline is a smooth curve passing through each of the sixteen points defining the ripple surface. Shown in Figure 1(ii) is the normalized tangential velocity amplitude at the ripple surface, which is further broken down into its horizontal and vertical components (for time $t = 0$). As expected, the magnitude of the horizontal velocity falls monotonically from crest to trough, and the vertical component is zero at the crest and trough positions. The maximum normalized tangential velocity occurs at the crest where $|u| = |u_{cr}| = 1.31 \ U$ and the minimum value occurs at the trough where $|u| = |u_{tr}| = 0.69 \ U$. These must be the extreme values of velocity anywhere in the flow domain since the motion is irrotational (Lamb, 1932, Art. 37). In Figure 2, vertical profiles of $|u/U|$ are plotted for the crest and trough positions. The curves shown take the extreme values quoted above at the bed surface, and both tend rapidly to unity (i.e. the unperturbed value) as $y/L$ increases.

At the crest and trough positions, the height of influence of the ripples has been taken as the distance from the bed surface to the level at which
It should be noted that this differs from the type of definition usually adopted in studies of frictional boundary layers. In the present example, the heights of influence are 0.69 \( L \) and 0.79 \( L \) at the crest and trough respectively.

It is important to note also that, although the zero-streamline depicted in Figure 1 passes through the sixteen points defining the ripple surface, small departures from a pure sine wave arise between these points. These departures, which have been evaluated through the range \( 0 \leq X \leq 0.5 \) at points spaced 0.005 apart, have been found generally to be very small and to vary with the choice made for \( Y_{a} \). In the present example, with the optimum choice \( Y_{a} = 0.15 \), the maximum difference anywhere over the ripple surface is 0.012\% of the ripple height. Smaller values of \( Y_{a} \) produce larger percentage discrepancies; for example, with \( Y_{a} = 0.10 \) the discrepancy is 0.082\%. On the other hand, larger values of \( Y_{a} (\geq 0.17) \) lead to a breakdown of the numerical method in that the matrix to be inverted for the doublet strengths becomes ill-conditioned.

The difference in volume per unit width between the exact and simulated ripple profiles is also very small. For \( Y_{a} = 0.15 \), this difference is \( 1.1 \times 10^{-8}\% \) of the volume of the exact sinusoidal shape (0.05 \( L^2 \)).

**Range of application of the solution.** To obtain a guide as to the range of application of the method in terms of ripple steepness \( H/L \) and shape \( S \), the general solution for complex potential (10) has been compared directly with results for the case of deep flow in a region defined by a conformal mapping suggested by Taylor, Gent and Keen (1976), namely

\[
z = \xi + \frac{iH}{2} \exp \left( ik \xi \right) \tag{13}
\]

where \( z = x + iy, \xi = \xi + i\eta \) and \( k = 2\pi/L \). This is similar to the mapping used by Benjamin (1959). If the bed surface \( (y = y_{b}) \) is defined as \( \eta = 0 \) then

\[
y_{b} = -\frac{H}{2} \cos k \xi \tag{14}
\]

For small \( H, x = \xi \) and the bed is purely sinusoidal in the original coordinates; however, for increasing values of \( H \), the ripple crests become progressively more and more peaked and the troughs flatter and longer. The ripples are symmetrical about their crests and the shape factor defined by (11) is given by

\[
S = 1 - 2 \left( H/L \right) \tag{15}
\]

The velocity components are

\[
(u, v) = (-J^{\frac{1}{2}} \phi_{\xi}, -J^{\frac{1}{2}} \phi_{\eta}) = (-J^{\frac{1}{2}} \psi_{\eta}, J^{\frac{1}{2}} \psi_{\xi}) \tag{16}
\]

where \( J \), the Jacobian of the transformation, is
Figure 3. Normalized flow velocities at the ripple surface: (i) at the crest, (ii) at the trough positions. A comparison of model results for bed shapes based on Eq. (14) with the known solution, Eq. (16). Also model results for a sinusoidal bed.

\[ J = \frac{\partial(\xi, \eta)}{\partial(x, y)} = \left\{ 1 - Hke^{-kn} \cos k\xi + \left( \frac{Hk}{2} e^{-kn} \right)^2 \right\}^{-1} \]  

A series of ripples defined by (14) has been examined and sixteen points per wavelength (equally spaced in \( x \)) have been taken from each for the purpose of defining the ripple profile. These points, together with a choice of \( Y_d \), have constituted the input information to the model. Solutions obtained from (16) and from the proposed model are compared in Figure 3, where the ratios \( |u_{cr}/U| \) and \( |u_{tr}/U| \) are plotted against ripple steepness. The values of the ratios increase at the crest and decrease at the trough as \( H/L \) increases. Also, by comparison with the model results for a sinusoidal bed which are shown in Figure 3, the ratios can be seen to be strongly, but separately, dependent on \( S \). In particular, the more peaked are the ripple crests (at a given \( H/L \)) the greater is the flow speed at the crest surface, and the shallower the troughs the smaller are departures of flow speed from the unperturbed value \( U \).

In comparing results from the proposed method with those from the solution given by (16), it can be seen that the values of \( |u_{tr}/U| \) are almost identical in the range of \( H/L \) shown (Fig. 3(ii)), whereas a systematic discrepancy develops in \( |u_{cr}/U| \) as \( H/L \) takes large values (Fig. 3(i)). This arises because the choice of sixteen specified points per ripple wavelength is insufficient to define the ripple profile adequately in the region of a crest which is strongly peaked. The discrepancy in Fig. 3(i) is large when \( S < 0.7 \), for example, when \( S = 0.6 \) (i.e. \( H/L = 0.2 \) from (15)), \( |u_{cr}/U| \) is 2.69 from Eq. (16) and 2.36 from the present model. The problem
of the definition of ripple profile does not arise in the region of the trough either for ripples defined by (14) or for sinusoidal profiles ($S = 1$). The troughs in both cases are simulated accurately by the model in the range of $H/L$ shown.

Where comparisons have been made between results for $|u_{cr}/U|$ for a given profile specified, firstly, by 8 and, secondly, by 16 coordinate points in each ripple wavelength, it has been found that an improvement in accuracy is achieved in the latter case and particularly so for small $S$. It might be expected, therefore, that the addition of further specified points in each ripple wavelength would reduce the discrepancy in $|u_{cr}/U|$ seen in Fig. 3(i). However it has not been thought necessary to proceed in this way since quite acceptable accuracy is achieved with 16 specified points over the range of $H/L$ of naturally occurring ripples on the seabed, in say $H/L < 0.15$. Allen (1968) quotes the range of steepness of natural “small-scale” ripples ($L < 60$ cm) as $0.05 < H/L < 0.2$, and of “large-scale” ripples ($L > 60$ cm) as $0.01 < H/L < 0.1$.

4. Results

The height of influence of ripples above the seabed. An important factor in the interpretation of velocity data is the thickness of the zone of influence of ripples above the seabed. The thickness of this layer, defined in Eq. (12), has been considered as the height locally above the bed.

In Figure 4, the thickness is shown at both crest and trough positions for various ripple types. Extending the comparisons of Section 3(ii), results for the solution corresponding to transformation (13) are compared with results from the present model for the range of values of steepness $0 < H/L < 0.2$. In addition, boundary zone thickness for a sinusoidal bed is shown, as well as results for naturally occurring ripples. For all these ripple types, the thickness ($\hat{y}$), can be seen to be less than the ripple wavelength ($L$), and to be smaller at the crest ($\approx 0.6 L$) than at the trough ($\approx 0.8 L$). Thus measurements made at heights above the bed greater than these can be regarded as representative of the unperturbed free stream flow. The thickness is weakly dependent on ripple height $H$, and also on shape $S$, in that the curves shown are not coincident in either Figure 4(i) or 4(ii).

The comparisons made with results based on transformation (13) show that the boundary zone thickness attains almost the same value by the two approaches (with a choice of sixteen doublets per ripple length in the model). In the case of the transformation, the nondimensional thickness $\hat{y}/L$ is given in the limit $H/L \to 0$ by

$$\exp \left\{ -2\pi \hat{y}/L \right\} = 0.01 \quad \text{or} \quad \hat{y}/L = 0.733$$

This limiting value of thickness at both crest and trough positions applies also to the sinusoidal bed, since (14) tends to a sine wave as $H/L$ becomes small.

The flow over natural ripples. From the results of Section 3 and Section 4(i) it is
Figure 4. Height of boundary influence (i) at the crest, (ii) at the trough positions. Results for the natural ripple shapes of Figure 5 have been included in (ii) only for those cases in which a "secondary crest" does not occur in a "primary trough".

It is clear that measurements of velocity made well above a rippled bed need to be interpreted carefully before an estimate of the flow velocity close to the bed surface can be made. It is clear also how apparent discrepancies can arise in such parameters as the sediment threshold velocity, when measurements are made in the free stream.
flow above rippled and flat beds; in particular, an apparent lowering of the threshold velocity is to be expected in the region of the ripple crest, and an apparent raising of the threshold in the trough. The former result has been found in the field experiments of Davies and Wilkinson (1978).

To seek a quantitative explanation for discrepancies of this type (see Section 1), measurements of real sand ripple profiles on the seabed have been made by divers and these shapes have then been used as an input to the present model. The shapes were obtained below the positions where velocity was monitored, by the careful insertion of thin rigid sheets into the seabed, on which profiles were drawn accurately over a full ripple length. Typical ripple shapes thus obtained are shown in Figure 5, and in Table 1 information is given about their principal dimensions including shape factor $S$. In the figure, the ripple profiles have been plotted for $0 \leq X \leq 1$ with their crest positions lying at the end points of this range. The profiles are somewhat
Table 1. The principal dimensions of the real ripples shown in Figure 5. Values of $S$ in parentheses indicate an estimate due to the presence of a “secondary crest” in the “primary trough”.

<table>
<thead>
<tr>
<th>Ripple number</th>
<th>Steepness $H/L$</th>
<th>Shape $S$</th>
<th>Wavelength $L$ (cms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.106</td>
<td>(0.62)</td>
<td>112.5</td>
</tr>
<tr>
<td>2</td>
<td>0.108</td>
<td>0.63</td>
<td>112.5</td>
</tr>
<tr>
<td>3</td>
<td>0.105</td>
<td>(0.70)</td>
<td>109.0</td>
</tr>
<tr>
<td>4</td>
<td>0.129</td>
<td>(0.75)</td>
<td>78.6</td>
</tr>
<tr>
<td>5</td>
<td>0.178</td>
<td>1.05</td>
<td>54.6</td>
</tr>
<tr>
<td>6</td>
<td>0.139</td>
<td>(0.60)</td>
<td>81.8</td>
</tr>
<tr>
<td>7</td>
<td>0.181</td>
<td>0.85</td>
<td>66.4</td>
</tr>
<tr>
<td>8</td>
<td>0.150</td>
<td>0.69</td>
<td>81.2</td>
</tr>
<tr>
<td>9</td>
<td>0.148</td>
<td>0.75</td>
<td>79.6</td>
</tr>
</tbody>
</table>

irregular and in certain cases have a “secondary crest” midway between the two principal crests; in cases of this kind, $S$ has been calculated by taking $H$ as the height from the ripple crest to the lowest point anywhere on the profile. The ripples are reasonably symmetrical about their troughs, and the sixteen coordinate points shown for each ripple have provided the input to the model. Passing a smooth zero streamline through these symmetrically placed points removes much unwanted fine detail, while it preserves the general shape of the features. With one exception the ripples have $S < 1$ reflecting their peaked crests and rather flatter troughs.

In Figure 6 results from the model are shown for certain of the ripple shapes in Figure 5. These vertical profiles of horizontal velocity show a much stronger enhancement of $|u|$ close to the surface of the ripple crest, than was seen in the case of the sinusoidal bed of steepness 0.1 (see Fig. 2). This can be explained mainly by reference to ripple shape $S$, although steepness $H/L$ is also a contributing factor.

Figure 6. Vertical profiles of normalized velocity for three of the ripples shown in Figure 5. The ordinate $Y$ is measured from the bed surface at the crest or trough as appropriate.
Figure 7. Normalized velocities at the surface of a ripple crest. Results are shown for the real ripples of Figure 5, the values by each point indicating ripple shape $S$. (Values in brackets are estimates of $S$ for ripples having a "secondary crest" in the "primary trough"). Model results are shown for Ripple #2 ($S = 0.63$) scaled to a general steepness $H/L$, and model results are shown also for a sinusoidal bed. In addition, a family of curves based on an extended use of transformation (13) is shown; these curves enable rough estimates of $|u_{cr}/U|$ to be obtained for symmetrical ripples, for given $H/L$ and $S$.

Above the main trough, the effect of "secondary crests" can be seen in two of the profiles.

In Figure 7, the single points show the surface values $|u_{cr}/U|$ for the ripple profiles in Figure 5, while the dashed line indicates results obtained by scaling Ripple #2 in such a way that its shape has been preserved while its steepness has been varied. This dashed curve has been used to provide values of $|u_{cr}/U|$ for the interpre-
tation of sediment threshold results in cases in which insufficient detail of the ripple shape was obtained for a reliable model run to be performed, but in which the ripple steepness was known. A case in point are the ripples discussed in Section 1 with $H/L = 1/7$ and for which a value of $|u_{cr}/U| = 1.86$ is obtained. This is close to the experimentally observed discrepancy of about 2 in the critical sediment threshold velocities on the rippled and flat beds. Results obtained with the present method, therefore, appear to offer a good quantitative explanation for the discrepancy. Corresponding values of $|u_{cr}/U|$ are complicated by the presence of “secondary crests” in the “primary ripple troughs” in several cases, as seen in Figure 6. No generally valid conclusions can therefore be deduced from these results.

Finally, referring to Figure 4, it can be seen that the heights of boundary influence are less above the crests of the natural ripples than above a sinusoidal bed, and that agreement is generally closer to the results obtained using transformation (13). It can be seen also that measurements made at heights greater than about $0.6L$ above the crests of naturally occurring ripples will be representative of the free stream flow conditions. Results for the trough positions are shown only for the cases in which “secondary crests” were not present in the “primary troughs.” The dashed line in Figure 4 for the representative ripple has been derived in an analogous way to that in Figure 7.

5. Discussion

In considering the results presented for the real ripple shapes in Figure 7 it is apparent that the calculated values of $|u_{cr}/U|$ depend strongly upon $S$. For example, two of the ripples (#8 and #9) with $H/L \approx 0.15$ have $|u_{cr}/U|$ equal to 1.65 and 1.90 respectively; this difference would appear to be accounted for by their values of $S$ of 0.75 and 0.69 respectively. Now it will be recalled that, in Section 3(ii), use was made of transformation (13) to evaluate the range of application of the proposed solution in terms of $H/L$ and $S$. In particular, it was shown that, as $H/L$ varies, transformation (13) leads to the single curve for $|u_{cr}/U|$ which is plotted in Figure 3, and along this curve the shape $S$ varies according to (15). The results for the real ripple shapes of Figure 5 have been considered in the light of this curve and it has been noticed that, on scaling $|u_{cr}/U|$ linearly by $H/L$ for each ripple, the point of intersection of the resulting line with the curve occurs at a value of $H/L$ where $S$ from Eq. (15) takes roughly the same value as that of the real ripple. This suggests the following simple approximate rule for obtaining rough estimates of $|u_{cr}/U|$ by an extended use of transformation (13), given a knowledge of $H/L$ and $S$ for a particular ripple; this involves taking the known value of $S$ and obtaining $|u_{cr}/U|$ from (16), and then scaling this value linearly by the ratio of the required steepness to the steepness given by (15). Alternatively, a graphical procedure can be adopted, as indicated by the family of curves drawn for discrete values of $S$ in Figure 7. It must be emphasized that transformation (13) provides only one point
on each of these curves, and that the linear behavior shown is not justified by an analytical argument. The limitations of the procedure outlined above are illustrated in Figure 7 by the results corresponding to the representative ripple with shape $S = 0.63$, which fit only roughly into the proposed empirical framework. Finally, it is suggested that rough estimates of the height of influence of ripples can also be made by an extended use of transformation (13), in particular by using an argument analogous to that given above which allows for both ripple steepness and shape.

6. Conclusions

In this paper, a model of the nonseparating oscillatory potential flow over a rippled bed has been developed. Although the flow has been assumed to be “deep,” the results presented are applicable if the ripple wavelength is less than the mean water depth. This restriction excludes using the model in its present form to study the flow over sand waves of length much greater than the water depth but, in principle, the existing formulation is capable of extension to this case. The fact that the flow has been assumed to be oscillatory is incidental to the model, and the results obtained for velocity amplitudes and boundary zone thicknesses can be used in steady flow problems without further assumptions—for example, in the problem of the steady flow over a pattern of symmetrical fossil ripples. If the bed features are evolving, the use of the model as it stands is more appropriate in the case of oscillatory flow than steady flow since, throughout this paper, the ripples have been assumed to be symmetrical about their crests. However, the potential solution can be generalized, if required, to study the flow over a pattern of ripples which are not symmetrical.

The model has been used to calculate the velocity field above sinusoidal ripples, above ripple shapes defined by a well-known conformal transformation and above real ripple shapes measured on the seabed. One of the main conclusions is that ripple steepness and ripple shape are separately important parameters in understanding the flow over a rippled bed; a shape factor $S$ has been proposed to characterize the peaked nature of ripple crests in relation to their flatter troughs. The thickness of the zone of influence of ripples above the bed has been calculated according to a given definition. This thickness has been shown to be weakly dependent upon both ripple shape and ripple height $H$, and to differ above the crest and trough; roughly, a value of $0.6L$ is appropriate at the crest and a value of $0.8L$ at the trough positions, where both heights are measured upward from the ripple surface and $L$ is the ripple wavelength. Also, in a direct application, results from the model have provided an explanation for an apparent discrepancy observed between the sediment threshold velocities on flat and naturally rippled sand beds.

The range of application of the solution has been assessed in terms of ripple steepness and shape by comparison with a known potential solution for the flow over a rippled bed of finite amplitude. It has thus been shown that good accuracy
is achieved by the method for the range of real ripple steepness found on the seabed.

Finally, it has been suggested that to provide rough estimates of the flows over naturally occurring ripple shapes, there is empirical support for an extended use of results obtained from the known potential solution mentioned above, if care is taken in interpreting the velocities thus obtained.

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