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Observations of the vertical distribution of low frequency kinetic energy in the Western North Atlantic

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ABSTRACT

Eddy or low frequency (periods greater than a day) kinetic energy per unit mass ($K_E$) in the western North Atlantic is observed to increase approaching the Gulf Stream system along 55W longitude from the interior of the subtropical gyre, by a factor of 30 in the thermocline and by over two orders of magnitude at 4000 m depth. $K_E$ at 4000 m depth decreases moving up the Continental Rise from the Gulf Stream. The relative vertical distribution of $K_E$ is found to be less depth dependent near the Gulf Stream (with increasing $K_E$?) than in the interior of the gyre. The vertical structure in the MODE-I region is an intermediate case.

The shape of the frequency spectrum for $K_E$ varies with depth and with geographical position; spectral estimates in the thermocline from the MODE-I area (28N, 70W) are dominated by longer time scales, but at a site near the Gulf Stream (37.5N, 55W), where spectral shape does not vary strongly in the vertical, the thermocline spectrum is more weighted toward the temporal mesoscale (~50-100 days) that dominates at 4000 m depth in the MODE-I area. Spectral estimates at depth appear to be weighted toward shorter time scales moving from the Gulf Stream up the Continental Rise, Slope, and Shelf. If the frequency spectrum for $K_E$ is divided into two pieces with periods less than 100 days in one and greater than 100 days in the second, then: (1) The relative vertical $K_E$ distribution near the Gulf Stream does not differ significantly from band to band, whereas in the MODE-I region the lower frequencies are much more depth dependent than the higher. (2) The relative vertical $K_E$ distribution for periods less than 100 days is not appreciably dissimilar at the two locations, although $K_E$ level and total vertical structure are distinct. (3) The relative vertical $K_E$ distribution at the lower frequencies in the MODE-I region does not differ significantly from the vertical distribution of total $K_E$ at a site farther into the interior of the gyre.

Time averaged currents at all depths where data are available show a relatively narrow (~200 km) and comparatively depth-independent return flow for the Gulf Stream system along 55W; transporting roughly $70 \times 10^6$ m$^3$ s$^{-1}$ westward, with about $25 \times 10^6$ m$^3$ s$^{-1}$ relative to the bottom and approximately $45 \times 10^6$ m$^3$ s$^{-1}$ associated with the "bottom transport."

1. Introduction

Evidence for an association of the properties of low frequency fluctuations (periods greater than one day) with the general circulation and geometry of the North Atlantic has been steadily accumulating (M. Swallow, 1961; Iselin, 1961; Crease,

1962; Fofonoff, 1967; Schmitz, Robinson, and Fuglister, 1970; J. C. Swallow, 1971; Dantzler, 1976, 1977; Freeland, Rhines, and Rossby, 1975; Wyrtki, Magaard, and Hager, 1976; Schmitz, 1976, 1977), and the observations described in the following support this general trend. Most previous investigations have been based on data of limited spatial and temporal duration and/or resolution. Long-term moored instrument data are now available over a wide range of depths at a variety of locations, allowing a qualitative description of the geographical variation of the vertical distribution of $K_B$ over a substantial area in the western North Atlantic. The nature of the long-term data now also permits a preliminary investigation of spatial variations in spectral shape.

The observations described below were the first direct measurements explicitly designed to examine the characteristics of both eddies and time-averaged currents in the westward return flow of the Gulf Stream system (a recirculation in close proximity with the Stream), a controversial region (Worthington, 1976; Schmitz, 1977; Stommel, Niiler, and Anati, 1978). Both the Stream and the return flow may be examined with long-term instruments at depths well below the thermocline and Schmitz (1976, 1977) has discussed some results from both regions at 4000 m depth (along 70W, 60W, and 55W). The return flow has recently become accessible up into the thermocline and new data in this region along 55W are discussed in the following. Although a westward return flow appears to be present at all longitudes and depths where data are available, the detailed configuration of this current is to be determined (note Figs. 2-5 in Stommel, Niiler, and Anati, 1978; and compare with Worthington, 1976; Schmitz, 1977; and Fig. 1 below). It is demonstrated in the following that the tendency for $K_B$ to increase along 55W approaching the return flow and Gulf Stream, previously documented at 4000 m and near surface depths only, also obtains at other depths up into the thermocline. Spectral shape in the thermocline shifts toward more energy at shorter time scales moving into the return flow and Gulf Stream from the interior of the subtropical gyre.

Eddy-resolving gyre-scale numerical models (Holland and Lin, 1975a,b; Holland, 1977; Robinson, Mintz, Harrison, and Sempter, 1977) have been developed in parallel with the acquisition of the more recent of the measurements noted above. Models and data both point to a strongly spatial inhomogeneous eddy field, with some qualitative agreement in the latitudinal distribution of $K_B$ and mean flow (Holland, 1977; Robinson, Mintz, Harrison, and Sempter, 1977).

A similar development of process-oriented numerical models (Bretherton and Karweit, 1975; Owens, 1975; Rhines, 1977) leads to the general qualitative premise that “open ocean” eddies should be predominantly zonal, less depth-dependent in more energetic regions, and more confined to the thermocline over rough topography (Rhines and Bretherton, 1973). Evidence is presented below in qualitative support of these conclusions.

Given the strong spatial inhomogeneity in properties of the eddy field, the obser-
vation that the relative vertical $K_B$ distribution for different frequency bands is nearly the same in widely varying locations might be especially significant. In particular, (1) fluctuations with periods less than 100 days have a similar relative vertical $K_B$ distribution in the MODE-I area to that near the Gulf Stream, over 1000 km distant, (2) fluctuations with periods greater than 100 days in the MODE-I area have a similar relative vertical distribution of $K_B$ to that of the total eddy field at a site over 1000 km farther into the interior gyre. These similarities in vertical structure might (this is conjecture) imply similarities in origin and/or dynamics.

The principal conclusions of this investigation have been noted in the Abstract and Introduction. Data and results are described in more detail in the following.

2. The data

Horizontal inhomogeneity of the low frequency variability in the western North Atlantic was suggested by the contrast in the data (M. Swallow, 1961; Crease, 1962; J. C. Swallow, 1971) between Aries Areas A and C, and by Fofonoff's (1967) comparison of current meter records from Site D with a record near the Gulf Stream. Mooring survival rates in the early 1960's precluded an extensive geographical exploration of the properties of the eddy field, and long-term moored instrument observations made by the Woods Hole Oceanographic Institution were concentrated (Fofonoff, 1968) at a single location (Site D; 39N, 70W), with a limited amount of exploration undertaken (Fofonoff and Webster, 1971), primarily along 70W. Moorings with subsurface flotation having a survival rate better than 90% for two to four month deployments became available in the late 1960's, initially limited to near bottom measurements. Baseline measurements south of the Site D along 70W were re-initiated. In 1969, comparatively energetic low frequency currents were observed in the vicinity of the Gulf Stream near 70W at a nominal depth of 4000 m (Schmitz, Robinson, and Fuglister, 1970), as was suggested earlier (Fofonoff, 1967; p. 100, Fig. 7). Measurements at 4000 m depth in the vicinity of the Gulf Stream along 70W were continued at intervals through 1974 (Luyten, 1977). Observations were started in 1971 in an area closer to the interior of the subtropical gyre (Gould, Schmitz, and Wunsch, 1974). In this region, centered at about 28N, 70W and called the MODE-I area, data were collected for about two years at two sites (MODE CENTER: 28N, 69.7W; and MODE EAST: 28.2N, 68.6W) at three depths (500, 1500, and 4000 m). Comparison of the measurements at 4000 m depth near the Gulf Stream along 70W and in the MODE-I region yielded $K_B$ estimates differing by an order of magnitude (roughly 10 cm$^2$ s$^{-2}$ at 28N and 100 cm$^2$ s$^{-2}$ near the Stream; Schmitz, 1976). Long-term SOFAR float measurements at 1500-2000 m depth yielded both zonal and meridional variations in $K_B$ along 28N between 68W and 73W and along 70W between 24N and 32N (Freeland, Rhines, and Rossby, 1975). The low frequency variability in the MODE-I area has been described in detail
Figure 1. Latitudinal distribution of time-averaged zonal current component at four depths along 55W. ○ denotes data from the first setting of POLYMODE Array 2 (roughly 7 month averages) and □ denotes data from a combination of the first and second deployments (about 17 data months). If significant data were lost, only the □ symbol is used. ▲ denotes space-time averages over three nine-month records from POLYMODE Array 1. Depths are underlined in each frame of the figure and the approximate location of the axis of the Gulf Stream is indicated.

elsewhere (Davis, 1975; Freeland, Rhines, and Rossby, 1975; Rhines, 1977; Richman, 1976; and Richman, Wunsch, and Hogg, 1977).

Exploratory measurements, called POLYMODE Array 1, were extended east of the MODE-I area to 55W in 1974 with nine-month deployments at 500, 1000, 2000, and 4000 m depths on several moorings. Another order of magnitude decrease in $K_B$ (∼1 cm² s⁻² at 28N, 55W) at 4000 m depth was found, and $K_B$ in the thermocline was observed to decrease by a factor of about 5 from 70W (50-60 cm² s⁻²) to 55W (∼10 cm² s⁻²) along 28N (Schmitz, 1976). The region near 28N, 55W,
is the least energetic site for which data are presently available in the western North Atlantic. Three moorings were deployed in this vicinity, permitting combined space-time averages for partially enhanced stability. In the following, this site will be denoted by the symbol $\triangle$.

The baseline measurements down 70W in the late 1960's and early 1970's were restricted primarily to 4000 m depth for logistical reasons. Tests at Site D and in the MODE-I area in the early 1970's had indicated that nine month to one year deployments up into the thermocline with a high survival rate could be feasible. Efforts after MODE-I and through POLYMODE Array 1 supported the premise, and increasing emphasis on quality control resulted in a significant improvement in current meter data return. Therefore, a two year deployment of twelve moorings, called POLYMODE Array 2, was planned to contain instruments up into the thermocline along 55W, moving as close to the Gulf Stream as considered feasible. A latitude of 37.5N was selected for the northernmost mooring site with instru-
ments up in the thermocline (hereafter, this site, the most energetic sampled at a variety of depths, will be referred to as PM08). Four moorings were set north of 37.5N under the Gulf Stream and into Slope Water at 41.5N with instruments at 4000 m depth only. The first setting of POLYMODE Array 2 occurred in April-May, 1975. This array was recovered and re-set in December, 1975 and again in October, 1976. All POLYMODE Array 2 moorings set to date have been recovered and data return has been about 90%. Data from 4000 m depth from the first setting have been described by Schmitz (1976). Data from the second setting are used in the following to describe some properties of the $K_B$ distribution in the thermocline as well as at depth.

The moorings, instruments, and techniques used to obtain and process the data discussed below have been described in some detail elsewhere (Fofonoff and Webster, 1971; Gould, Schmitz, and Wunsch, 1974; Heinmiller and Walden, 1973; Reinmiller, Schmitz, and Briscoe, 1974; Heinmiller, 1976).

### 3. Mean zonal flow

One way to examine the stability of time averages is to compare results over a variety of averaging intervals. Figures 1 and 2 contain averages over both the first setting of POLYMODE Array 2 (7 months, nominal) and the combined first and second settings (17 months, nominal). In Figures 1 and 2 a bar is used to display an approximate range of positions of the axis of the Gulf Stream, as indicated by the position of the $15^\circC$ isotherm at 200 m depth (Fuglister and Voorhis, 1965), located between 39.5 and 40.5N at 55W according to Schroeder (1963). The mean flow is generally more stable at depth than in the thermocline and Figure 1 looks better at 600 m than it probably should because the 36N, 55W data are available only from the first setting of the array. However, data are available for the second setting of the array at 600 m depth at a mooring 40 km to the east of the mooring at 36N, 55W. The mean zonal speed there is observed to be $-13.9 \text{ cm s}^{-1}$, in comparison with $-14.6 \text{ cm s}^{-1}$ from the first setting at 36N, 55W shown in Figure 1.

Time-averaged or mean zonal current components plotted in Figure 1 exhibit a comparatively narrow (~200 km) westward return flow at all depths in the vicinity of 36N along 55W. This current is rather weakly depth-dependent, especially at depth. A very crude estimate (linear fits above and below the thermocline for the mooring at 36N, 55W only) of the time-averaged transport of this current yields $70 \times 10^8 \text{ m}^3 \text{ s}^{-1}$ (40 above 1250 m depth and 30 below), a vertically averaged time-mean zonal speed of 14 cm s$^{-1}$. Of the 70, 25 is relative to the bottom and 45 associated with the “bottom reference level” (~9 cm s$^{-1}$ in 5 km depth for a current ~200 km wide). The vertically extrapolated, time-averaged, surface zonal speed obtained by this procedure is approximately 22 cm s$^{-1}$. 

4. Energy levels

\( K_B \) at 4000 m depth was observed to intensify approaching the Gulf Stream system from 28N along both 70W and 55W by Schmitz (1976, 1977). According to Freeland, Rhines, and Rossby (1975; Fig. 6), \( K_B \) at 1500-2000 m depth increases both north and south from the MODE-I area along 70W. POLYMOMODE Array 2 results permit examination of \( K_B \) variations (Fig. 2) at depths up into the thermocline from the interior of the subtropical gyre through and just north of the return flow (Fig. 1) along 55W. \( K_B \) in Figure 2 increases approaching the Gulf Stream at all depths where data are available. This increase is over two orders of magnitude (1 \( \rightarrow \) 150 cm\(^3\) s\(^{-2}\)) below the thermocline and roughly a factor of 30 (9 \( \rightarrow \) 290 cm\(^2\) s\(^{-2}\)) in the thermocline. Schmitz (1976) also noted an order of magnitude increase in thermocline \( K_B \) up 60W from 28N to 31-34N. Estimates of \( K_B \) for the surface layers along 55W by Wyrtki, Magaard, and Hager (1976) vary much less (400 \( \rightarrow \) 600 cm\(^2\) s\(^{-2}\)), relatively, between 28 \( \rightarrow \) 38N. Eddy-resolving gyre-scale numerical models tend to predict (qualitatively) the observed \( K_B \) fall-off moving away from the Gulf Stream, at least at depth, as does Flierl (1977) for the radiation field of Gulf Stream Rings. The difference between \( K_B \) in the thermocline and at 4000 m depth (\( \delta K_B \)) along 55W intensifies from about 10 cm\(^2\) s\(^{-2}\) at 28N to nearly 180 cm\(^2\) s\(^{-2}\) at 37.5N, and Dantzler (1977) reports an increase in baroclinic potential energy per unit mass (\( P_B \)) from about 100 \( \rightarrow \) 1600 cm\(^2\) s\(^{-2}\) at thermocline depths along 55W from 28 \( \rightarrow \) 38N, roughly the same relative increase.

In addition to amplification moving north into the Gulf Stream system out of the interior of the subtropical gyre, \( K_B \) has been observed to grow moving westward toward the continental boundary along 28N at a variety of depths. Freeland, Rhines, and Rossby (1975) observed \( K_B \) at 1500 m depth to increase from 4 to 8 cm\(^2\) s\(^{-2}\) moving westward from the Bermuda Rise out over the Hatteras Abyssal Plain at MODE-I latitudes. Schmitz (1976) noted that \( K_B \) at \( \Delta \) (28N, 55W) was less than 1 cm\(^2\) s\(^{-2}\) at 4000 m, as opposed to about 10 cm\(^2\) s\(^{-2}\) in the MODE-I area (28N, 70W). \( K_B \) in the thermocline intensifies from about 10 cm\(^2\) s\(^{-2}\) at \( \Delta \) to 50-60 cm\(^2\) s\(^{-2}\) at MODE CENTER. In contrast, according to Wyrtki, Magaard, and Hager (1976), the surface \( K_B \) is essentially the same (\( \sim 400 \) cm\(^2\) s\(^{-2}\)) at the two sites as is \( P_B \) in the thermocline (\( \sim 100 \) cm\(^2\) s\(^{-2}\)) reported by Dantzler (1977). \( \delta K_B \) changes from \( \sim 10 \) cm\(^2\) s\(^{-2}\) at \( \Delta \) to 40-50 cm\(^2\) s\(^{-2}\) at MODE CENTER, not in step with the distribution of \( P_B \), contrary to the case along 55W.

The Aries measurements should not be taken to be indicative of an energetic eddy field throughout the ocean interior, nor should the MODE-I observations. The Aries data were obtained in the vicinity of the return flow, and \( K_B \) at 4000 m depth there is over twice \( K_B \) in the thermocline in the interior of the subtropical gyre at \( \Delta \). \( K_B \) in the MODE-I area is a factor of 5 to 10 that at \( \Delta \). All evidence points to a very inhomogeneous field. Recent numerical models (Holland and Lin, 1975a,b; Holland, 1977; Robinson, Mintz, Harrison, and Sempter, 1977) are characterized...
Figure 3. Estimates of the decadal frequency (ν) spectrum for the total $K_B[K_B(T)]$ at MODE CENTER, from depths (m) in parentheses.

by strong eddy-mean flow interactions in the Gulf Stream and return flow, and possibly other special locations, but not in the interior of the subtropical gyres. The data presented here and in Schmitz (1977) tend to support this result, in that eddies are more energetic and have stronger gradients in their statistics in these regions. It seems possible that the downstream enhancement in transport of the Gulf Stream might be eddy-driven. The transport through the Straits of Florida (Schmitz and Richardson, 1968) is close (Schmitz, 1977) to estimates of the wind-driven transport for the North Atlantic (Evenson and Veronis, 1975; Saunders, 1976). Leetmaa, Niiler, and Stommel (1977) and Niiler, Simco, and Larue (1971) have presented evidence that the mid-Atlantic transport is in line with the Sverdrup balance.

The kinetic energy per unit mass associated with periods less than one day or with the internal wave frequency band ($K_I$) is generally much less inhomogeneous in the data from POLYMODE Arrays 1 and 2 than is $K_B$ (Wunsch, 1976; Schmitz, 1977). Approaching the northern section of the Array 2 full depth moorings, $K_I$ jumps rather abruptly by a factor of 2 at 600 m depth (nominal). Overall, $K_I$ varies by a factor of two to three at all depths over the same geographical range where $K_B$ varies by one to two orders of magnitude.

5. Time scales

In addition to vertical and horizontal variability in energy level, low frequency fluctuations in the western North Atlantic will be shown also to be spatially inhomogeneous in estimates of the shape of their decadal (energy or area preserving) kinetic energy frequency spectrum. There are only a few sites where sufficiently long
Figure 4. Estimates of the decadal frequency ($\nu$) spectrum for the total $K_B[T](T)$ in the thermocline at MODE CENTER and at a site near the Gulf Stream (PM08; 37.5N, 55W), from depths (m) in parentheses.

data series are available in both the thermocline and in the deep water. The two extremes in energy level are the MODE-I area and the northernmost thermocline mooring of POLYMODE Array 2 (PM08).

The earliest exploration of the low frequency variability in the MODE-I area suggested that the deep currents there were dominated by fluctuations with periods in the range of 50-150 days (Gould, Schmitz, Wunsch, 1974) as had been inferred for the Aries data (Crease, 1962; Swallow, 1971). With over two years of data now available, the decadal kinetic energy spectral estimate for 4000 m depth at MODE CENTER yields a well-defined temporal mesoscale (Fig. 3), in contrast to the case for the thermocline, where decadal $K_B$ spectral estimates increase with decreasing frequency. As indicated in Figure 3, the two ranges of time scales are tentatively labeled “mesoscale,” the dominant signal at 4000 m; and “secular scale,” the most
pronounced signal in the thermocline. In all spectral figures, the full bandwidth of the estimates is brought out by plotting in the form of bar graphs. If the frequency is of the order of the bandwidth, conventional estimates of the decadal spectrum are too high. In Figures 3-7, this bias is significant only for the lowest frequency band, and none of the conclusions drawn here are affected qualitatively. The spectral content of low frequency currents along 28N has been discussed in more detail elsewhere (Owens, 1975; Freeland, Rhines, and Rossby, 1975; Richman, 1976; Richman, Wunsch, and Hogg, 1977). Numerical model runs analyzed by Owens (1975; Fig. 18, p. 56) reproduce the 4000 m MODE CENTER spectral estimate in Figure 3 quantitatively.

It should not, however, be assumed that longer time scales obscure the mesoscale in the thermocline everywhere. For example, the temporal mesoscale is quite prominent in the thermocline near the Gulf Stream (PM08) along 55W (Fig. 4). The thermocline spectral shape at PM08 is somewhat similar to that from 4000 m at MODE CENTER (Fig. 5), as is the spectrum for 4000 m at PM08 (Fig. 6).

Along 70W, there is a general tendency for a relative shift in energy toward higher frequencies at depths below the thermocline moving up the Continental Rise from the Gulf Stream (Luyten, 1977) and also up the continental slope (Schmitz,
1974), and possibly continuing onto the continental shelf (Beardsley, Boicourt, and Hansen, 1976). There is some evidence for this trend in the deep POLYMODE Array 2 data (Fig. 7; PM12 is the site indicator for moorings at 41.5N, 55W).

Figure 6. Estimates of the decadal frequency ($\nu$) spectrum for the total $K_{E}[K_{E}(T)]$ at indicated sites, from depths in parentheses (m). Different ordinate scales are associated with the site identifier above the respective ordinates.

Figure 7. Estimates of the decadal frequency ($\nu$) spectrum for the total $K_{E}[K_{E}(T)]$ at indicated sites, from depths in parentheses (m).
6. Vertical structure

Vertical distributions of $K_E$ (Fig. 8) at three sites spanning the observed range of geographical variation of $K_E$ may also be compared in terms of their relative or normalized vertical structure (Fig. 9). The relative vertical $K_E$ distributions in Figure 9 are less depth dependent with increasing energy level (approaching the Gulf Stream system), a qualitative observation in possible support of general theoretical arguments by Rhines (1977). In addition, $\Delta$ is over rough topography, whereas MODE CENTER would be characterized as comparatively smooth. Also, the zonal eddy kinetic energy tends to be larger than the meridional along 28N, by a factor of about two in the thermocline for both $\Delta$ and MODE CENTER.

Figure 8. Vertical distribution of $K_E$ at three sites in the western North Atlantic. Numbers in parentheses are differences in $K_E$ between the thermocline and 4 km depth.

Figure 9. Normalized or relative distributions of $K_E$ for the same three sites used in Figure 8. Numbers in parentheses next to the site identifier are crude estimates of vertically integrated $K_E$. 
The differences in spectral shape between MODE CENTER and PM08 in the thermocline also imply different relative vertical $K_E$ distributions for different frequency bands. If $K_E$ is split between periods greater or lower than 100 days, then the relative $K_E$ distribution (Fig. 10a) for the mesoscale is not significantly different between the two sites. That is, mesoscale eddies have essentially the same relative vertical $K_E$ distribution in the MODE-I area as they do near the Gulf Stream along 55W, despite the difference in energy level (a factor of about 15 in the thermocline). Lower frequencies (Fig. 10b) are more depth dependent at MODE CENTER, but at PM08 both ranges of time scale have nearly the same vertical $K_E$ distribution. Energy levels at these two sites for the longer time scales in the thermocline are not drastically different (only a factor of 2 or so).
Figure 11. Relative vertical distributions of $K_E$ for indicated sites, with two different normalizations. The values for MODE CENTER are from periods greater than 100 days. Numbers in parentheses next to the site identifier are crude estimates of vertically integrated $K_E(K_T)$.

The relative vertical $K_E$ distribution for the lower frequencies at MODE CENTER is similar to the relative vertical distribution of total $K_E$ at $\Delta$ (Fig. 11). This result holds whether one normalizes by $K_E$ at 4000 m depth (Fig. 11a) or by a crude estimate ($K_T$) of vertically averaged $K_E$ (Fig. 11b). That is, the relative vertical structure at $\Delta$ is approximately like that at MODE CENTER, with the mesoscale eddies removed from the latter site.

The relative vertical structures at MODE CENTER for the two frequency bands reported here are somewhat similar to the two empirical vertical modes described by Davis (1975; Fig. 9, p. 26).

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