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Tide-induced residual current, a 2-D nonlinear numerical tidal model

by K. T. Tee¹,²

ABSTRACT

Tide-induced residual current was studied through solution of the two-dimensional nonlinear shallow-water equation. A simple numerical method was developed and applied to Minas Channel and Minas Basin at the head of the Bay of Fundy where strong residual currents of up to 0.76 m/sec have been measured. Two models are considered: the first is rectangular, of constant depth, and has a shape similar to the Minas Channel and Minas Basin, and the second approximates the actual geometry of the area studied. The numerical solutions are stable under various conditions and produce four strong eddies in the residual current. The results obtained with the models are strongly supported by the observations available.

A test indicates that the residual currents are induced by the tidal current through the inertial effect. In terms of vorticity, the residual eddies result from the advection of vorticity generated in a boundary layer.

1. Introduction

Residual currents are the net direction and amplitude of water movement after all the sinusoidal tidal currents have been removed. They can be generated by wind stress, density gradients in the ocean, or rectification of the sinusoidal tidal currents. The latter method of generation is the concern of this paper.

Asymmetric tidal currents (and hence residual currents) have often been measured in channels, shallow seas, and bays, and near sandbanks (Caston and Stride, 1970; Klein, 1970; Hunter, 1972; Robinson, 1966; and Inshore Tides and Current Group, B.I.O., 1966). In the Minas Channel and Minas Basin at the head of the Bay of Fundy, residual currents with a maximum velocity of 0.76 m/sec in Minas Passage have been recorded (Inshore Tides and Current Group, B.I.O., 1966, see also Figs. 1 and 8), as well as the world record tide range of up to 15 m and tidal current of 5.6 m/sec (Cameron, 1961). These extreme tides are thought to be due to resonance near the $M_2$ frequency of the Bay of Fundy and Gulf of Maine (Garrett, 1972, 1974).

Residual currents can be either Eulerian or Lagrangian. The Eulerian residual is

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the residual current at a fixed point in space, whereas the Lagrangian residual, equal
to the Eulerian residual plus the Stokes drift, gives the residual motion of a fluid
element. In the following discussion, unless otherwise specified, “residual current”
will refer to the Eulerian residual current.

Tides may generate residual currents through: (i) nonlinear bottom friction, (ii)
the nonlinear terms in the continuity equation, and (iii) the nonlinear advective
terms in the momentum equation. Many studies (e.g. Ramming, 1972) ignore (iii)
although in many areas, such as Minas Channel and Minas Basin, the strong tidal
currents and complicated geometry can produce strong inertial effects that dominate
in the generation of the residual current. In this paper, the hydrodynamic equations
used will include all these nonlinear terms.

In most numerical tidal models, certain simplifications are made to ease the com-
putation. For example, only a few authors have taken advection into consideration.
Leendertse (1967) used a multi-operational method in his nonlinear model. Flather
and Heaps (1975) introduced a finite difference technique to compute the $M_2$ tide
in Morecambe Bay. They employed the “angle derivative” approach to represent
the nonlinear advective terms in the equation of motion. Residual currents were not
calculated in these models. Hunter (1972) developed a two-dimensional nonlinear
tidal model and calculated the residual currents induced by both meteorological and
tidal inputs. Unfortunately, the eddy viscosity coefficient he used had to be fifty times larger than the expected value in order to ensure numerical stability of his computation. Including the “tidal stress” in the equation of motion, NiHoul and Ronday (1975) computed the Lagrangian residual current in the southern bight of the North Sea. The relative importance of the Eulerian residual and Stokes drift was not discussed. In all the models, the mechanisms that generate tide-induced residual currents were not studied.

One- and two-dimensional numerical models of the Bay of Fundy have been formulated (Yuen, 1969; Greenberg, 1969; Heaps and Greenberg, 1974; Duff, 1970; Parkinson, 1972) as part of studies designed to estimate the effects of a proposed tidal barrier on the tides in the Bay. The use of one-dimensional models in the Minas Channel and Minas Basin is incomplete because of its complicated geometry. Some of the two-dimensional models did not include the Minas Basin in their calculation. All the models neglected the advective terms in the momentum equations.

In Section 2, a simple and straightforward numerical technique for solving the full hydrodynamical equations is discussed.

2. Numerical model

a. Hydrodynamical equation. The model is based on the following equation:

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\mathbf{f} \times \mathbf{u} - g \nabla \zeta + A_h \nabla^2 \mathbf{u} - \gamma \frac{\mathbf{u} \cdot \mathbf{n}}{H} \]

\[
\frac{\partial \zeta}{\partial t} + \nabla \cdot (H \mathbf{u}) = 0
\]

where \(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\) is the horizontal Laplacian operator; \(\zeta\) is the height of water surface above mean sea level; \(\mathbf{u}\) is the horizontal velocity vector with components \(u\) and \(v\) in a cartesian system with coordinates \(x\) and \(y\); \(|\mathbf{u}| = (u^2 + v^2)^{\frac{1}{2}}\) is the magnitude of vector \(\mathbf{u}\); \(H = \zeta + D\) is the total depth of the water column, where \(D\) is the depth of the bottom below mean sea level; \(g\) is gravity; and \(\mathbf{f}\) is the Coriolis parameter. Bottom friction is represented by a quadratic term with coefficient \(\gamma\) and horizontal mixing processes by a constant eddy viscosity coefficient \(A_h\).

The tide-generating force is neglected in this model because \(\nabla \zeta_0\) is of the order of \(10^{-8}\), where \(\zeta_0\) is the semidiurnal equilibrium tide, whereas \(\nabla \zeta\) is of the order of \(10^{-5}\) (see Fig. 9).

b. Finite difference formulation. In rewriting equation (1) in finite difference form for numerical computation, a continuous function \(B(x, y, t)\) representing \(u, v,\) or \(\zeta\) is written in the following notation:

\[
B_{i,j}^n = B(x = i\Delta x, y = j\Delta y, t = n\Delta t)
\]
where $i,j,n$ are integers. Central difference approximation is used in formulating the derivative numerically,

$$\frac{\partial}{\partial a} B(a) = \frac{B(a+\Delta a) - B(a-\Delta a)}{2\Delta a}$$

where $a$ represents $x$, $y$, or $t$.

The distribution of grid points is shown in Fig. 2. The velocity components $u$ and $v$ are computed at even $n$ and the elevation $\xi$ is computed at odd $n$. The grid spacings $\Delta x$ and $\Delta y$ are chosen to be constant and set $\Delta T = 2\Delta t$ and $\Delta s = 2\Delta x = 2\Delta y$. To calculate $u$ and $v$ at $\xi$ points or vice-versa, the following spatial average is used:

$$B_{i,j} = \frac{1}{4} (B_{i,j+1} + B_{i,j-1} + B_{i+1,j} + B_{i-1,j})$$

The map of Minas Channel and Minas Basin is taken from Canadian Hydrographic Chart D7-4010. The scaling problem due to the Mercator projection is ignored because the area studied is deformed by only $10^{-8}$. 

Figure 2. Map showing the computational scheme, the rigid boundary (---) and open boundary (-- -- --). (• ) points for $u$ and $v$, (x) points for $\xi$. 
The finite difference formulation of equation (1) for even \( n \) is:

\[
\frac{u_{i,j}^{n+1} - u_{i,j}^{n-1}}{\Delta T} = -fx u_{i,j}^n - (u \cdot \nabla u)_{i,j}^n - (g \nabla \zeta)_{i,j}^n
\]

\[+ (A_h \nabla^2 u)_{i,j}^{n-1} - \frac{\gamma}{H_{i,j}^n} (|u|u)_{i,j}^{n-1}
\]

\[
\frac{\zeta_{i+1,j}^{n+2} - \zeta_{i+1,j}^{n}}{\Delta T} = - (\nabla \cdot Hu)_{i+1,j}^{n+1}
\]  

(2)

The Coriolis terms at time step \( n \) are approximated by an average between the values at time steps \((n+1)\) and \((n-1)\)

\[f u_{i,j}^n = f \frac{u_{i,j}^{n+1} + u_{i,j}^{n-1}}{2}
\]

(3)

This choice of semi-implicit form is to ensure the neutral stability of the system (Crowley, 1970). Substituting equation (3) into equation (2), the momentum equation can be written as

\[u_{i,j}^{n+1} = \frac{1}{1 + F^2} (X_{i,j} + FY_{i,j}), \quad V_{i,j}^{n+1} = \frac{1}{1 + F^2} (Y_{i,j} - FX_{i,j})
\]

(4)

where

\[F = -\frac{f}{2} \Delta T
\]

\[X_{i,j} = u_{i,j}^{n-1} + F V_{i,j}^{n-1} + \Delta T (-u_{i,j}^n \frac{u_{i+1,j}^{n+1} - u_{i-1,j}^{n-1}}{\Delta s}
\]

\[- \frac{\nu_{i,j}^n u_{i,j+1}^{n+1} - u_{i,j-1}^{n-1}}{\Delta s} - g \frac{\zeta_{i+1,j}^{n+1} - \zeta_{i-1,j}^{n-1}}{\Delta s}
\]

\[+ A_h \frac{u_{i+2,j}^{n+1} - u_{i-2,j}^{n-1} + u_{i+1,j+2}^{n+1} - u_{i-1,j-2}^{n-1} - 4u_{i,j}^{n-1}}{\Delta s^2}
\]

\[- \gamma \frac{|u|_{i,j}^{n-1} u_{i,j}^{n-1}}{H_{i,j}^n}
\]

and similarly for \( Y_{i,j} \).

The forward time step is used for \( u \) in the bottom friction terms. The finite difference forms of these terms is discussed in the Appendix. It is found that the damping rates of the system for explicit and semi-implicit formulations of the friction terms are the same and equal to the analytical value, and the shifts in frequency due to the finite difference forms of the equations are small and can be neglected. By changing \( \beta \) in the Appendix to \( A_h \left( \frac{\Delta T}{\Delta s^2} \right) (2 - \cos k \Delta s - \cos l \Delta s) \), the same results can be obtained for the viscous terms.
The finite difference form of the viscous terms given in equation (4) decouples the grid points into two separate groups, such as $u_{i,j}$, $u_{i+1,j}$, $u_{i,j+1}$, $u_{i,j+2}$, $u_{i+2,j+2}$, etc. and $u_{i+1,j+1}$, $u_{i+3,j+1}$, $u_{i+1,j+3}$, $u_{i+1,j-1}$, etc. Another finite difference form of the viscous terms

$$(A_h \nabla u)_{i,j} = A_h \frac{u_{i+1,j+1}^{n-1} + u_{i-1,j+1}^{n-1} + u_{i+1,j-1}^{n-1} + u_{i+1,j-1}^{n-1} - 4u_{i,j}^{n-1}}{0.5 \Delta s^2}$$

(5)

was developed to test the sensitivity of the results to different representations of the viscous terms. The computation was performed with $A_h = 10^2 \text{m sec}^{-1}$ in the numerical model of the Minas Channel and Minas Basin. (see Fig. 14 for the model). The calculated circulation pattern of the residual current did not change significantly.

A central time step is used in the advective terms. The $u^n$ in equation (4) is obtained by using the Taylor series expansion:

$$u_{i,j}^n = u_{i,j}^{n-1} + \left( \frac{\partial u}{\partial t} \right)_{i,j}^{n-1} \frac{\Delta T}{2} + \frac{1}{2!} \left( \frac{\partial^2 u}{\partial t^2} \right)_{i,j}^{n-1} \left( \frac{\Delta T}{2} \right)^2 + \ldots$$

(6)

Using the typical values of $\left| \frac{\partial u}{\partial t} \right| \sim 10^{-4} \text{m/sec}^2$, $\Delta T \sim 31 \text{sec}$, and $|u| \sim 1.5 \text{m/sec}$, it is found that the second and higher order of $\left( \frac{\Delta T}{2} \right)$ terms in equation (6) can be neglected. Equation (6) is then written as:

$$u_{i,j}^n = u_{i,j}^{n-1} + F v_{i,j}^{n-1} + \frac{\Delta T}{2} \left( - u_{i,j}^{n-1} \frac{u_{i+1,j}^{n-1} - u_{i-1,j}^{n-1}}{\Delta s} - v_{i,j}^{n-1} \frac{u_{i+1,j}^{n-1} - u_{i-1,j}^{n-1}}{\Delta s} - g \frac{\xi_{i+1,j}^n - \xi_{i-1,j}^n}{\Delta s} + A_h \frac{u_{i+1,j}^{n-1} + u_{i-1,j}^{n-1} + u_{i,j+1}^{n-1} + u_{i,j-1}^{n-1} - 4u_{i,j}^{n-1}}{\Delta s^2} \right)$$

(7)

$$- \gamma \frac{|u|_{i,j}^{n-1} u_{i,j}^{n-1}}{H_{i,j}^n}$$

and similarly for $v_{i,j}^n$.

This method of computing $u^n$ was introduced by Lax and Wendroff (1960) and has been extensively used and developed (i.e. Fischer, 1965). This study differs from others in applying this technique: specifically $u^n$ is computed at $(u,v)$ points instead of $(\zeta)$ points. This difference was found to improve the neutral stability of the system (see section d).

To estimate $\zeta^{n+1}$ in the continuity equation, the Lax-Wendroff method is also applied;
The total depth $H_{i+1,j}^{n+1}$ is equal to $D_{i+1,j} + \zeta_{i+1,j}^{n+1}$. $\zeta^{n+2}$ can then be calculated from the continuity equation,

$$\zeta_{i+1,j}^{n+2} = \zeta_{i+1,j}^{n} - \Delta T \left( \frac{H_{i+2,j}^{n+1} u_{i+2,j}^{n+1} - H_{i,j}^{n} u_{i,j}^{n+1}}{\Delta s} \right)$$

$$+ \frac{H_{i+1,j+1}^{n+1} v_{i+1,j+1}^{n+1} - H_{i+1,j-1}^{n+1} v_{i+1,j-1}^{n+1}}{\Delta s}$$

Equations (4), (7), (8) and (9) are iterated in cyclic order to obtain $u$ and $\zeta$ in the next time step.

c. Initial and boundary conditions. The initial values of $\zeta$ are calculated from

$$\zeta = \zeta_0 \cos (G)$$

where $\zeta_0$ denotes the observed $M_2$ amplitude and $G$ denotes the $M_2$ phase lag. For the second model of the Minas Channel and Minas Basin, $\zeta_0$ and $G$ are taken from the co-tidal charts prepared by Godin (1968). For the rectangular models described in section 3, $\zeta_0$ and $G$ are taken from the co-tidal chart shown in Fig. 3. This chart is prepared in such a way that the values of $\zeta_0$ and $G$ resemble the tidal regime in the Minas Channel and Minas Basin. The initial values of $u$ and $v$ are taken as zero. The shoreline is designed to pass through the $(u,v)$ points. An example is shown in Fig. 2. At a shoreline, the depth vanishes and $u$ and $v$ are set equal to zero. This condition is equivalent to introducing a boundary layer of a thickness on the order of grid spacing (2.8 km), which may be an overestimate. This is compensated for by situating the rigid boundary inside the coastline. An example is shown in Figs. 7 and 14. It will be shown that in these two cases there are no significant differences in the circulation patterns of the residual current in the area studied. For the $M_2$ tidal current, the model shown in Fig. 7 gives results that compare better with the observed values.
Along the open boundary, the tidal elevation $\zeta$ is prescribed,

$$\zeta^n = \zeta_0 \cos (n \omega \Delta t - G)$$

(10)

where $\omega$ is the $M_2$ angular speed. This boundary condition is not sufficient for our model because the computation of advective terms at the grid point just inside the open boundary requires knowledge of the current at the entrance. One of these current components, $u$ or $v$, can be calculated by using the continuity equation at the entrance. For example, from the equation at grid point $(I,J)$ in Fig. 2,

$$\left( \frac{\partial \zeta}{\partial t} \right)_{I,J}^n = - \frac{(H_{I+1,J,n} u_{I+1,J,n-1} - H_{I-1,J,n} u_{I-1,J,n-1})}{\Delta s}$$

$$- \frac{(H_{I,J+1,n} v_{I,J+1,n-1} - H_{I,J-1,n} v_{I,J-1,n-1})}{\Delta s}$$

(11)

$v_{I,J-1}$ can be calculated as

$$v_{I,J-1}^{n-1} = \frac{1}{H_{I,J-1}^{n-1}} \left[ \left( \frac{\partial \zeta}{\partial t} \right)_{I,J}^n \Delta s + H_{I+1,J,n} u_{I+1,J,n-1} + H_{I,J+1,n} v_{I,J+1,n-1} \right]^{-1}$$

(12)

where \( \left( \frac{\partial \zeta}{\partial t} \right)_{I,J}^n = \omega (\zeta_0)_{I,J} \sin (\omega n \Delta t - G_{I,J}) \). To obtain $H_{I,J-1}^{n} = \zeta_{I,J-1}^{n} + D_{I,J-1}$ in equation 12, $\zeta_{I,J-1}^{n}$ is calculated from the observed values of $(\zeta_0)_{I,J-1}$ and $G_{I,J-1}$,

$$\zeta_{I,J-1}^{n} = (\zeta_0)_{I,J-1} \cos (\omega n \Delta t - G_{I,J-1})$$

(13)

The uncertainty associated with the estimate of $\zeta_{I,J-1}^{n}$ is not important in the calculation of $H_{I,J-1}^{n}$ because $D_{I,J-1} \sim 10 \cdot (\zeta_0)_{I,J-1}$.

To estimate $u_{I,J-1}^{n}$, we take the direction of $u$ at the entrance as always parallel to the shoreline. This direction of $u$ is assumed true because the major axis of the current near the entrance to Minas Channel has been observed to be in this direction (Inshore Tides and Current Group, B.I.O., 1966). Thus

$$u_{I,J-1}^{n} = v_{I,J-1}^{n} \tan \theta$$

(14)

where $\theta$ is the angle between the shoreline and the $y$-axis. After obtaining $u$ and $v$ at point $(I,J-1)$, $u$ and $v$ at $(I+1, J-2)$ (Fig. 2) can then be calculated by using the same procedure. Repeatedly applying this method, all the $(u,v)$ points at the entrance can be determined.

As no experimental data at the entrance are available, the prescribed tidal elevation and estimated current in this area introduce uncertainty in the open boundary conditions. How this uncertainty affects the results in the inner areas was tested by changing the estimated current at the entrance from (12) and (14) to $u (I,J-1) =$
\( \mathbf{u}(I,J+1), \mathbf{u}(I+1,J-2) = \mathbf{u}(I+1,J) \ldots \) etc. (see Fig. 2), such that the current stays the same in the \( y \)-direction; this is simply due to the fact that the shoreline is almost parallel to the \( y \)-axis. The residual current near the entrance changed slightly in the northern part and very significantly in the southern part. These changes decrease from the entrance and become insignificant beyond the Cape D'Or area. Errors arising from the uncertainty in the open boundary may also be indicated near the entrance by the strong \( M_4 \), \( M_6 \) and \( M_8 \) current, and strong minor axes of \( M_2 \) current. These currents have been calculated in the numerical model of the Minas Channel and Minas Basin shown in Fig. 7, both with a dam either in the Cape Blomidon or Burntcoat Head area, and without a dam. It was found that although they are strong at the entrance, they weaken toward the inner portion of the channel and become weak in the Cape D'Or area. Hence the errors arising from the uncertainty of the open boundary conditions are probably insignificant beyond the Cape D'Or area.

The main contribution to the advective terms \( \mathbf{u} \cdot \nabla \mathbf{u} \) is from the \( M_2 \) current. Thus, although the residual current at the entrance may not be parallel to the coast, the error in the advective terms is small because the major axis of \( M_2 \) current is always in this direction.

The computer calculations begin at \( n = 0 \) with the initial values described above, and are allowed to continue for a number of equivalent \( M_2 \) tidal cycles until the computed values settle down with sufficient accuracy to a steady state.

d. Stability problem. To study the neutral stability of our numerical method, equation (1) is linearized and all the friction terms are excluded. For simplicity, the stability calculation is performed by first excluding the advective terms and then including this term in a one-dimensional case. This analysis provides a good indication of neutral stability in the numerical method only and does not guarantee the stability of the solution in the complete model.

Using the grid scheme shown in Fig. 2 and the numerical method described in the previous sections, Miyazaki (1965) obtained for the case without advection the following neutral stability criterion:

\[
\Delta T \leq \frac{\Delta s}{\sqrt{2gH_m}}
\]

where \( H_m \) is the maximum depth of water. This is the standard Courant-Friedrich-Lewy stability condition for a gravity wave.

To study the stability problem arising from the advective terms \( (\mathbf{u} \cdot \nabla \mathbf{u}) \) we consider the following one-dimensional equation:

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} = 0
\]

(16)
where $U$ is characteristic velocity, taken to be constant. Applying the same procedure used in the Appendix, and the computational technique described in the previous section, we obtain

\[(1 - \alpha^2 (1 - \cos 4p) - i2\alpha \sin 2p - \lambda). \quad (1 - \lambda) - \left(-i \frac{D\Delta T}{\Delta s} 2 \sin p \sqrt{\lambda}\right).\]

\[
\left[-\alpha \left(\frac{g\Delta t}{\Delta s}\right) (\cos p - \cos 3p) - i \left(\frac{g\Delta T}{\Delta s}\right) 2 \sin p\right] \sqrt{\lambda} = 0
\]

where

\[
\alpha = \frac{U\Delta T}{2\Delta s}
\]

\[
p = k\Delta x
\]

$\lambda$ is an amplification factor defined as $e^{\sigma\Delta t}$ (see Appendix). For neutral stability we require $|\lambda| = 1$ for any value of $k$. For $|\lambda| < 1$, although the solution is stable, the numerical method produces damping in the system, which is improper in our calculation because the damping terms $\left(-\gamma \frac{|u|u}{H}\right)$ and $A_h \nabla^2 u$ were not included in (16). The method is unstable for $|\lambda| > 1$.

Let $q = \frac{gD\Delta T^2}{\Delta s^2}$ (17) can be written as

\[
\lambda^2 + (A + B i) \lambda + (P + Q i) = 0
\]

where

\[
A = -2 + \alpha^2 (1 - \cos 4p) + 4 q \sin^2 p
\]

\[
B = 2\alpha \sin 2p - \alpha q (\cos p - \cos 3p) 2 \sin p
\]

\[
P = 1 - \alpha^2 (1 - \cos 4p)
\]

and

\[
Q = -2\alpha \sin 2p
\]

$\lambda$ can then be solved as

\[
\lambda_1 = \left(-\frac{A}{2} - \frac{1}{2\sqrt{2}} \right) (R + (R^2 + I^2)\frac{1}{2}) + i \left(-\frac{B}{2} + \frac{1}{2\sqrt{2}} \right) \frac{I}{(R + (R^2 + I^2)\frac{1}{2})}
\]

\[
\lambda_2 = \left(-\frac{A}{2} - \frac{1}{2\sqrt{2}} \right) (R + (R^2 + I^2)\frac{1}{2}) + i \left(-\frac{B}{2} - \frac{1}{2\sqrt{2}} \right) \frac{I}{(R + (R^2 + I^2)\frac{1}{2})}
\]

where

\[
R = A^2 - B^2 - 4p
\]

\[
I = 2A B - 4Q
\]

Taking $D = 60$ m, $\Delta s = 2830$ m, and $\Delta T \sim 31.04$ sec used in our model, $\lambda_1$ and $\lambda_2$ can be calculated for various values of $U$ and $p$. By changing $U$ from 0.15 m/sec to 1.5 m/sec and $p$ from 0 to $2\pi$, it was found that for all the values of $p$ (0 to $2\pi$), $|\lambda_1|$ and $|\lambda_2|$ vary from:
\[ \sim 1 \pm 10^{-6} \text{ for } U = 0.15 \text{ m/sec} \]

to
\[ \sim 1 \pm 10^{-7} \text{ for } U = 1.5 \text{ m/sec} \]

This deviation of \(|\lambda|\) from unity is, of course, very small compared to the damping terms, which are of the order of \(10^{-4}\) for \(U = 0.15 \text{ m/sec}\) and \(10^{-5}\) for \(U = 1.5 \text{ m/sec}\) (see Appendix). Thus our numerical method produces more or less neutral stability in the solution.

Applying the same technique used in the Appendix, the frequency \(\omega\) can be calculated. By using the typical value of \(\omega \sim 10^{-4}\) in our model (e.g. \(M_2, M_4\)), it was found that the frequency shift due to the finite different form of the advective term is small (\(\sim 0.1\%\)).

If \(u^n\) in equation (7) was calculated at \((\zeta)\) points, the deviation of \(|\lambda|\) from unity would vary from \(-10^{-6}\) for \(U = 0.15 \text{ m/sec}\) to \(-10^{-4}\) for \(U = 1.5 \text{ m/sec}\); this deviation is three orders of magnitude greater than in our solution.

e. Parameter values. Equation (15) gives the maximum time step that can be used in numerical modelling of gravity waves. In our numerical computation \(\Delta s = 2830 \text{ m}\) and \(H_m = 80 \text{ m}\); thus \(\Delta T \leq 71 \text{ sec}\). By changing \(\Delta T\) in equations (19) and (20), the effect of advective terms on the maximum \(\Delta T\) that can be used in our numerical model was tested. It was found that including the advective terms reduces \(\Delta T\) by only \(0.02\%\). In the following calculation, \(\Delta T \sim 31.04 \text{ sec}\) is used. This small value of \(\Delta T\) is chosen to improve the accuracy of the numerical solution.

The Coriolis parameter, \(f = 1.037 \times 10^{-4}\) sec\(^{-1}\), is chosen to be constant in the calculation. For the bottom friction coefficient \(\gamma\) and eddy viscosity coefficient, \(A_h\), the exact values in shallow water are not known. \(\gamma\) has always been taken as 0.002 to 0.003 in tidal computation (Dronkers, 1964). The value of \(A_h\) depends on the space resolution in the calculation. A test was conducted in our model with \(\gamma = 0.001\) and \(0.003\) and \(A_h = 0, 1, 10\) and \(10^2\) m\(^2\)/sec. It will be shown in the next section that the general circulation pattern of the residual current does not show any significant difference with various values of \(\gamma\) and \(A_h\). In most of the following calculations \(\gamma = 0.003\) and \(A_h = 10^2\) m\(^2\)/sec is used.

3. Results

The (Eulerian) residual current \(u_R\) is calculated by integrating the current \(u\) over \(M_2\) period \(T\),
\[
\begin{align*}
\mathbf{u}_R &= \frac{1}{T} \int_0^T \mathbf{u} \, dt \\
&= \frac{1}{N} \sum_{i=1}^{N} \mathbf{u}
\end{align*}
\]  
and is represented in finite difference form as
\[ \mathbf{u}_R = \frac{1}{N} \sum_{i=1}^{N} \mathbf{u} \]
where $N = T/\Delta T$. In our calculation, $N$ is equal to 1440. As $N$ is so large that the difference in results calculated by equations (21) and (22) can be expected to be very small. Using a sinusoidal current, it is found that this difference is $5 \times 10^{-6}$ of the tidal current for $N = 1440$. Since the residual current calculated in our model is of the order of 1 to 10% of tidal current, the difference can be ignored.

The net momentum per unit width $M_n$ can be calculated as:

$$M_n = \frac{1}{T} \int_0^T \mathbf{u} \cdot (D + \zeta) \, dt$$

or

$$U_i = u_R + U_s$$

where $U_i = \frac{M_n}{D}$ is the Lagrangian residual current and $U_s = \frac{1}{TD} \int_0^T \mathbf{u} \zeta \, dt$ is the Stokes drift.

1. Simple models. It was stated in the introduction that the residual currents in Minas Channel and Minas Basin are very strong. The particular features of the topography in this area are the intrusions of Cape Split, Cape Blomidon, and Cape D'Or areas from the smooth coastline (Fig. 1). To test the numerical method, a simple model resembling the shape of this region was devised. Fig. 4 shows a simple rectangular model with the shape ABCDEF representing Cape Split and Cape Blomidon. The parameters used in this model are $\gamma = 0.003, A_k = 10^5 \text{ m}^2 \text{ sec}^{-1}$, $\zeta = 0.1 \text{ m/sec}$.
and $D = 60$ m. The tide and tidal current are computed through eight tidal cycles. It is found that the model is stable and a steady state is reached such that there are no significant changes of the tidal motion and residual current from the 7th to the 8th cycle.

The calculated residual current is shown in Fig. 4. Four eddies are observed in this residual current. The eddies in areas I and III are clockwise and those in areas II and IV are counter-clockwise. The amplitude of this residual current is a few centimeters per second, which is 1 to 10% of the maximum tidal current.

By extending the rigid and open boundaries to a greater distance (Figs. 5 and 6), in order to pump more water into area I, the tidal current is increased to a maximum of about 1.5 m/sec in areas II and III. The residual current is found to be stronger. The significant result is that the circulation pattern of residual currents in area I, II, III, and IV stays the same. The shape GHIJ in Fig. 6 represents Cape D'Or.

To test the numerical method and to examine the changes of residual current with different values of the coefficients in the equation of motion, the computation was performed with $\gamma$ equal to 0.001 and 0.003, and $A_h$ equal to 0, 1, 10, and $10^2$ m$^2$ sec$^{-1}$. The model is stable for these values of $\gamma$ and $A_h$. Figures 6b and 6c show the calculated residual current with $\gamma = 0.001$ and $A_h = 10^2$ m$^2$ sec$^{-1}$, and $\gamma = 0.003$ and $A_h = 0$. The conclusions are: (1) that there is no significant change of the circulation pattern in the residual current for different values of $\gamma$ and $A_h$ used and (2) that the magnitude of residual current ($u_R$) is generally increased by reducing the values of $A_h$ and $\gamma$. This increase of residual current is due to the reduced damping of vorticity generated in the boundary layer (see Section 4).

b. Numerical model of Minas Channel and Minas Basin. For a more precise comparison of the calculated residual current with observation, a model including the actual depth of water and the details of the shoreline of Minas Channel and Minas Basin is used. The parameters are $\gamma = 0.003$ and $A_h = 10^2$ m$^2$ sec$^{-1}$.
Figure 6. Pattern of the residual current in the simple model. The open boundary KL in Fig. 5 has been removed to a greater distance. The shape GHIJ represents Cape D'Or (a) $A_h = 10^2 \text{ m}^2 \text{sec}^{-1}$, $\gamma = 0.003$; (b) $A_h = 10^2 \text{ m}^2 \text{sec}^{-1}$, $\gamma = 0.001$; (c) $A_h = 0$, $\gamma = 0.003$.

b.1. Eulerian residual current. The calculated residual current is shown in Fig. 7. The important feature of this circulation pattern is that the four eddies found in areas I, II, III, and IV in the simple models (Figs. 4, 5, and 6) were reproduced in this model. The magnitude of this residual current is very large (of the order of 0.5 m/sec in several areas (Fig. 7)).

Current meter data in the Bay of Fundy were collected in 1960 and 1965. The data were analysed by Godin (1968) and the Inshore Tides and Current Group, B.I.O. (1966) and the resulting residual current is shown in Fig. 8. Meteorological influence can be neglected in this study as strong residual current is observed even in calm weather (Godin, 1968). Comparing the results in Figs. 7 and 8 we can see that the numerical calculation reproduces the observed residual currents well. In area IV, the counter-clockwise circulation of residual current (Fig. 7) is clearly indicated by the four observed values at stations (75), (1), (2), and (76). The exist-
ence of the other three eddies, in the clockwise direction in areas I and III, and in the counter-clockwise direction in area II, are also supported by the observed values at stations (74), (H), and (B). The detailed comparison of these values is given in Table 1. We can see that they are in good agreement. At station 73, the experimental results are unreliable because of the uncertainty about the frame of reference in the 1965 observation (Godin, 1968). Table 2 compares calculated and observed $M_2$ tidal currents at the stations in the Minas Channel and Minas Basin occupied during the 1960 and 1965 measurements. It can be seen that the observed $M_2$ tidal currents are also well reproduced.

Figs. 9 and 10 show the distributions of calculated amplitude and phase of surface elevation. We can see that tides observed at coastal stations are reproduced well within 5 cm of amplitude and 5° of phase.

Near the entrance to the Minas Channel, Fig. 7 shows that the residual current flows north. This northward flow is consistent with Godin's (1968) prediction.

Further current meter data were collected in the Minas Basin in 1974 in an experiment to check the present prediction, and will be reported elsewhere.

---

Figure 7. Pattern of the residual current in the numerical model of Minas Channel and Minas Basin. $A_0 = 10^2$ m$^2$ sec$^{-1}$; $\gamma = 0.003$ and $D =$ actual depth of water.

Figure 8. The observed residual current in Minas Channel and Minas Basin. The integers associated with (•) are the station numbers. (Ref. to Inshore Tide and Current Group, B.I.O., 1966 and Godin, 1968).
b.2. Stokes drift. The circulation pattern of the Stokes drift is shown in Fig. 11. In most of the area, the Stokes drift, of the order of 0.05 m/sec (~10% of \( u_R \)), is directed toward the head of the Bay. This is because the energy flux \( g \bar{\xi} u \) is directed in this direction. In fact, the integral of \( gD \cdot \) Stokes drift across any given section must equal the energy flux into the system. Thus, a net outward Eulerian residual current is implied but it is only a small part of the total residual current (e.g. ~4% in the Minas Passage). A clockwise eddy is observed in area I, which is in the same direction as the residual current (Fig. 7).

To understand what causes this circulating Stokes drift, let us divide \( u \) and \( \xi \) into:

\[
 u = u_R + u' \quad \text{and} \quad \xi = \xi_R + \xi'
\]

where \( \xi_R \) is the residual in tidal elevation, and \( \xi' \) and \( u' \) are the sinusoidal parts of the tide and tidal current. From (24) we obtain:

Table 2. Comparison of observed and calculated \( M_2 \) tidal currents*

<table>
<thead>
<tr>
<th>Station Numbers (Fig. 9)</th>
<th>Major (m sec(^{-1}))</th>
<th>Minor (m sec(^{-1}))</th>
<th>Inclination to True North (deg)</th>
<th>Greenwich Phase Lag (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>Observed</td>
<td>Calculated</td>
<td>Observed</td>
</tr>
<tr>
<td>72</td>
<td>1.82</td>
<td>2.09</td>
<td>0.06</td>
<td>0.08</td>
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<td>1.34</td>
<td>1.32</td>
<td>0.03</td>
<td>0.13</td>
</tr>
<tr>
<td>74</td>
<td>2.11</td>
<td>1.68</td>
<td>0.03</td>
<td>0.11</td>
</tr>
<tr>
<td>75</td>
<td>1.22</td>
<td>1.57</td>
<td>0.19</td>
<td>0.06</td>
</tr>
<tr>
<td>76</td>
<td>0.85</td>
<td>0.77</td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td>2</td>
<td>1.46</td>
<td>1.54</td>
<td>0.31</td>
<td>—</td>
</tr>
<tr>
<td>H</td>
<td>2.28</td>
<td>2.21</td>
<td>0.05</td>
<td>—</td>
</tr>
<tr>
<td>B</td>
<td>0.80</td>
<td>1.08</td>
<td>0.11</td>
<td>—</td>
</tr>
</tbody>
</table>

* see Godin (1968) and Inshore Tide and Current Group (1966).
1976]  
*Tee: Tide-induced residual current, a model*  
619

Figure 9. (Upper). The calculated $M_2$ co-amplitude lines in Minas Channel and Minas Basin. Amplitude (m) ------. The underlined numbers are observed values of $M_2$ amplitudes. $A_h = 10^9$ m$^2$ sec$^{-1}$ and $\gamma = 0.003$.

Figure 10. (Lower). The calculated $M_2$ co-phase lines in Minas Channel and Minas Basin. Phase (deg.) ------. The underlined numbers are observed values of $M_2$ phase. $A_h = 10^9$ m$^2$ sec$^{-1}$ and $\gamma = 0.003$.

$$U_s = \frac{u_R \zeta_R}{D} + \frac{1}{TD} \int_0^T (u' \zeta') \, dt$$  \hspace{1cm} (25)

Except at the head of the Basin, $\zeta_R/D$ is always less than 1%, thus, the first term on the right hand side of (25) is unimportant and is neglected in the following discussion. The second term's value is due mostly to the $M_2$ component. Contribution from other constituents, $M_4$, $M_6$ and $M_8$, are negligible, and, of course, cross terms are zero.

The Stokes drift due to $M_2$ tidal elevation (amplitude $\zeta_{20}$, phase $g_2$) and $M_2$ current (amplitude $U_{20}$, phase $G_2$) in the direction of major axis is:

$$U_s \text{(Major)} = \frac{\zeta_{20} U_{20} \text{(Major)}}{2D} \cos (G_2 - g_2)$$

Similarly,

$$U_s \text{(Minor)} = \frac{\zeta_{20} U_{20} \text{(Minor)}}{2D} \sin (G_2 - g_2)$$

To determine the direction of $U_s$ (Minor), we rotate 90° from the flood direction according to the circulation of $M_2$ current.

The figures of $G_2$ and $g_2$ are shown in Figs. 10 and 12. For $90^\circ > G_2 - g_2 > -90^\circ$, Stokes drift is in the flood direction, (e.g. points A, B, C, D in Fig. 11), and will be in the ebb direction for $270^\circ > G_2 - g_2 > 90^\circ$ (e.g. point E in Fig. 11).
Figure 11. (top) Pattern of the Stokes drift. \( A_s = 10^3 \text{ m}^2 \text{ sec}^{-3}, \gamma = 0.003 \) and \( D = \text{actual depth of water} \).

Figure 12. (second) The calculated Greenwich phase lags of \( M_z \) tidal current \( A_s = 10^2 \text{ m}^2 \text{ sec}^{-3}, \gamma = 0.003 \) and \( D = \text{actual depth of water} \). The underlined numbers are the observed values.

Figure 13. (third) Pattern of the Lagrangian residual current. \( A_s = 10^2 \text{ m}^2 \text{ sec}^{-3}, \gamma = 0.003 \) and \( D = \text{actual depth of water} \).

Figure 14. (bottom) Pattern of the residual current in Minas Channel and Minas Basin. The width of Minas Passage has been narrowed. \( A_s = 10^3 \text{ m}^2 \text{ sec}^{-3}, \gamma = 0.003 \) and \( D = \text{actual depth of water} \).
For $G_2 - g_2 \sim 90^\circ$ or $270^\circ$, Stokes drift will depend on the magnitude of the minor axis, and becomes very small if this axis is very small (e.g. point F in Fig. 11). $U_s$ (Minor) becomes important in area I where $(G_2 - g_2)$ is close to $90^\circ$ and the minor axis is 20 to 25% of the major axis. For example, at point G in Fig. 11, $G_2 - g_2 \sim 89^\circ$, $U_{20}$ (Minor)/$U_{20}$ (Major) $\sim 23\%$, and the rotation of $M_2$ current is clockwise; calculation of Stokes drift will give the $U_s$ shown in Fig. 11.

b.3. Lagrangian residual current. The circulation pattern of Lagrangian residual current is shown in Fig. 13. Because the Eulerian residual current is much larger than the Stokes drift in most of the area, or both are in the same direction (clockwise eddy in area I), the circulation pattern of Lagrangian residual current in areas I, II, III, and IV does not show any significant difference from the Eulerian one.

4. Discussion

Including advective terms in our calculation shows that the tidal amplitude is reduced by only 2 to 4% in the Minas Basin, but their effect on the current is very significant as will be seen in the following discussion.

In Minas Passage, the eddies in areas II and III are small. However, these eddies are considered to be real because each eddy extends over about 12 grid squares, and the results are verified from the experimental observation at stations 74 and H (see Figs. 7 and 8).

To further study the existence of the eddies in areas I, II, III, and IV, a test was run to examine the sensitivity of the residual current to the spacing in Minas Passage. Fig. 14 shows the residual current for a narrow cross section in the Minas Passage. We can see that the circulation pattern of the residual current stays the same.

The model shown in Fig. 14 is used to study the residual current in the following two cases: (i) the area of the Minas Basin is reduced by excluding the mud flats (dotted areas in Fig. 1) and (ii) the depth of mean sea level $D$ is set constant and equal to 60 m. It was found that the circulation pattern of residual current in areas I, II, III, and IV produced in these cases does not differ significantly from that shown in Fig. 14. This result indicates the small influence of the irregular bottom topography and the distribution of mud flats in Minas Basin on the eddies in the residual current. Thus, it is probable that the eddies in the Minas Channel and Minas Basin, as in many others, arise from the particular coastal geometry.

Effects of friction on residual currents have been studied in section 3 in the simple model. It was found that the residual current increases as the effect of these friction terms decreases (see Fig. 6). Thus friction is not the dominant mechanism generating the residual current in the area studied.

To examine the cause of the residual current from other mechanisms, we simplify the momentum equation by
Figure 15. Comparison of $u_c/u_{cf}$ in various conditions at positions L, M, N, O, P and Q (see Fig. 14). The values of $u_c/u_{cf}$ at points 1, 2, 3 and 4 along the horizontal axis are calculated by (i) neglecting Coriolis terms; (ii) neglecting advective terms; (iii) neglecting advective and Coriolis terms; and (iv) neglecting advective and Coriolis terms, and changing friction terms to $-cu$, where $c = 7.5 \times 10^{-6}$ sec$^{-1}$.

(i) neglecting the Coriolis terms ($f \times u$); 
(ii) neglecting the advective terms ($u \cdot \nabla u$); 
(iii) neglecting the advective and Coriolis terms ($u \cdot \nabla u$ and $f \times u$); 
(iv) neglecting the advective and Coriolis terms, and changing the friction terms to $-cu$, where $c = r \frac{|u|}{H} \sim 7.5 \times 10^{-5}$ sec$^{-1}$ for $\gamma = 0.003$, $|u| = 1.5$ m sec$^{-1}$, and $H = 60$ m.

$A_h = 10^2$ m$^2$ sec$^{-1}$ and the actual depth of water are used in these calculations. The strength of the residual currents can be indicated by $u_c = \frac{|u_R|}{|u_{max}|}$, where $|u_R|$ and $|u_{max}|$ are the speeds of residual current and the maximum tidal current. Let $u_{cf}$ be the values of $u_c$ calculated by using the full equation of motions. The
plot of \( u_c/u_{ct} \) at positions L, M, N, O, P, and Q (see Fig. 14) in various conditions is shown in Fig. 15. These positions are situated in areas I, II, III, and IV (Fig. 14) where the four strong eddies in the computed residual current are located. Thus the values of \( u_c/u_{ct} \) at these positions indicate the variation of the strength of these eddies. Comparing \( u_c/u_{ct} \) at points 1, 2, 3, and 4 (Fig. 15), we can see that neglecting the Coriolis force and linearizing the friction terms changes the strength of the residual current by only a few per cent. The most drastic change in the residual current is caused by neglecting the advective terms. The values of \( u_R \) are reduced from the order of 0.5 m/sec to few centimeters per second. From \( u_c/u_{ct} \) at point 2, we can see that the strength of the residual current is reduced by more than 90%. This result strongly indicates that the residual current is generated by the inertial effect.

During flood tide, water enters Minas Channel from the Bay outside, is forced through the Minas Passage, and then enters the Minas Basin as a jet. In order to pump a large volume of water into the Basin to raise the sea surface by about 10 m from low tide to high tide, the current has to be very strong in the Minas Passage. A maximum current of up to 5.6 m/sec has been measured (Cameron, 1961). As the water jets into the Basin, the current from the Minas Passage produces a strong inertial force that keeps the current in area A larger than that in area B (Fig. 16). The inflow to the Minas Channel (Fig. 16) is more or less uniform as was expected. During ebb tide, the water is expelled from the Minas Basin through the Minas Passage and the current is expected to be more or less uniform in the Basin (areas A and B in Fig. 17). The inertial effect again produces stronger currents in area C than in area D in the Minas Channel (Fig. 17). As the same amount of water that was pumped into the basin at flood tide is pumped out at ebb tide, the strength of the uniform current is expected to be between the values of strong and weak current. Thus, after integrating the current over a tidal period, there will be a residual current in the flood direction at A and D, and in the ebb direction at B and C. The result is to form in the residual current a clockwise eddy in the Minas Basin and a counter-clockwise eddy in the Minas Channel.

It is important to note that the formation of eddies in the residual current requires a source of vorticity. In this study the eddies can be explained through the advection of vorticity \( \omega' = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \) generated in a boundary layer. From (1) the vorticity equation can be written as

\[
\frac{\partial \omega'}{\partial t} + u \cdot \nabla \omega' = -f \nabla \cdot u + A_h \nabla^2 \omega' - \nabla \times \frac{|u||u|}{H}
\]  

(26)

As the strength of residual current is changed by only a few per cent if the Coriolis force is neglected and the friction terms linearized, (26) can be approximated by

\[
\frac{\partial \omega'}{\partial t} + u \cdot \nabla \omega' = A_h \nabla^2 \omega' - c \omega'
\]  

(27)
where $c$ is the linear friction coefficient. The time average of (27), denoted by $\langle - \rangle$, is

$$\mathbf{u} \cdot \nabla \omega' = A_h \nabla^2 \bar{\omega}' - c\bar{\omega}'$$

(28)

This equation indicates that the net amount of vorticity advected into the region is equal to the dissipation of steady vorticity formed by the residual current. For the long wave theory used in the tidal computation, the advection of vorticity generated in the bottom boundary layer is neglected. Uncertainty in the open boundary condition may result in the advection of vorticity generated at the entrance. However, the effect of this uncertainty has been shown in the previous section to be insignificant in studying the circulation pattern of residual current in areas I, II, III, and IV. Thus, the important source of vorticity in this study is that generated in a rigid boundary. It is then very important that the nonslip boundary condition should be applied. Otherwise, this important source of vorticity will be eliminated. This boundary condition was not used in the earlier studies of residual current (Rampling, 1972; Hunter, 1972; NiHoul and Ronday, 1975).

The eddies in residual current can be a result of boundary layer separation. As the Reynold number in this study is large because of the strong current ($\sim 1 \text{ m/sec}$) and small diffusion constant ($\sim 10^2 \text{ m}^2 \text{ sec}^{-2}$), there will be some streamlines leaving the boundary layer and advecting vorticity to the open area.

During the flood tide in the Minas Channel and Minas Basin, anti-clockwise
vorticity is generated along the coast in the northern part and clockwise vorticity is generated in the southern part. This vorticity changes signs as the current reverses direction from flood to ebb, or vice versa. Some of it is advected by the tidal current into the open area. For example, the clockwise vorticity is advected during flood tide around point a into area I, and around point b into area III (Fig. 16). During ebb tide, the anticlockwise vorticity is advected around point c into area II and around point b into area IV (Fig. 17). Thus, there is a sink of clockwise vorticity in areas I and III, and a sink of anticlockwise vorticity in areas II and IV. A residual circulation is thus formed clockwise in areas I and III and anticlockwise in areas II and IV (Fig. 14).

There are other eddies in the residual current that also result from the advection of vorticity. For example, advection of clockwise vorticity around points d and e during ebb tide results in the clockwise eddies in areas V and VI (Figs. 17, 14). These eddies are not considered in this study because: (i) no experimental data are available for comparison, and (ii) the residual current and the eddies are either too small or subject to the uncertainty in the open boundary conditions.

In the steady state, the net amount of vorticity accumulated during a tidal cycle has to be balanced by damping and diffusion of steady vorticity formed by the residual current (equation 28). In a large tide, the net amount of vorticity accumulated is large because more vorticity is advected into the area. In order to dissipate this large amount of vorticity, the steady vorticity has to be large. Thus, there will be a strong residual current for a large tide. An example is the increase of residual current by increasing the tidal current (see Figs. 4 and 5).

Reducing the values of \( \gamma \) or \( A_h \) or both will make the current stronger, which will generate more vorticity along the boundary layer and thus more vorticity will be advected into the open area. As the friction is reduced by decreasing \( A_h \) and \( \gamma \), or both, it is expected that the steady vorticity has to increase. Thus the residual current will increase by decreasing \( A_h \) or \( \gamma \), or both (see Fig. 6).

It is important to note that the total vorticity generated within a boundary layer is explicitly independent of its thickness and depends only on the current at the outer edge of the layer. As measured currents are reproduced well in this study (Table 2), our numerical model, although it may not resolve the thickness of the boundary layer properly, can provide the correct estimate of the vorticity, and thus of the residual current. This result also explains why the poor resolution in the Minas Passage does not affect the circulation pattern of the residual current.

5. Conclusions

The residual currents calculated from the two-dimensional nonlinear numerical tidal models are found to have one counter-clockwise eddy in Minas Channel, one clockwise eddy in Minas Basin, and two eddies, one clockwise and the other counter-clockwise, in Minas Passage. The magnitude of these residual currents is
very strong, of the order of 0.5 m/sec in several areas. These results compare well with the observations available.

Numerical experiments were made to study the mechanism generating these residual currents. It was found that they were induced by the strong tidal current through the inertial effect and were a result of advecting vorticity generated in a boundary layer.

The model also reproduces well the observed tide and tidal current in the area studied.

Acknowledgment. I am grateful to Dr. C. J. R. Garrett for his valuable advice and criticism. I thank Drs. C. Quon, A. J. Bowen and K. Bryan for many useful discussions. The financial assistance provided by the Izaak Walton Killam Fund is gratefully acknowledged.

APPENDIX. Damping Rate of Bottom Friction

To study the effect on the damping rate for explicit and semi-implicit formulation of the friction terms, we consider the following two-dimensional linear equation

\[\frac{\partial M}{\partial t} = -gD \frac{\partial \zeta}{\partial x} - cM\]
\[\frac{\partial N}{\partial t} = -gD \frac{\partial \zeta}{\partial y} - cN\]
\[\frac{\partial \zeta}{\partial t} = -\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\]  
\[(A1)\]

where \(M = Du\) and \(N = Dv\) is the volume transport per unit width and \(c\) is the linear friction coefficient. For the typical solution of the form

\[W = We^{(\sigma t + ikx + ily)}\]  
\[(A2)\]

where \(W\) is a vector-valued function with components \((M, N, \zeta)\), \(\sigma\) is a frequency, and \(k\) and \(l\) are wave numbers in \(x\) and \(y\) directions, the dispersion relation of \((A1)\) (Kagan, 1971) is

\[\sigma_1 = c, \sigma_{2,3} = -\frac{c}{2} \pm i \left(k^2gD - \frac{c^2}{4}\right)^{\frac{1}{2}}\]  
\[(A3)\]

where \(k^2 = k_x^2 + k_y^2\). The oscillation frequency \(\omega\) is

\[\omega = \pm \left(k^2gD - \frac{c^2}{4}\right)^{\frac{1}{2}}\]  
\[(A4)\]

To examine the change of amplitude due to damping, we define \(\lambda = e^{\omega\Delta t}\) with modulus

\[|\lambda| = \frac{|\zeta|}{|\zeta(\text{zero})|} = \frac{|u|}{|u(\text{zero})|}\]  
\[(A5)\]

where \(\zeta\) (zero) and \(u\) (zero) denotes the value of \(\zeta\) and \(u\) calculated with zero bottom friction terms. In order to distinguish the values of \(\lambda\) calculated for different cases, we set \(\lambda^e\), \(\lambda^s\) and \(\lambda^t\) respectively as the values of \(\lambda\) for analytical solution, and for the finite difference solutions with explicit and semi-implicit formulation of the friction terms. Using equation \((A3)\), \(|\lambda^s|\) can be calculated as

\[|\lambda_1^s| = e^{-\varepsilon\Delta t} \sim 1 - 2\beta, \quad |\lambda_3^s| = e^{-\varepsilon\Delta t} \sim 1 - \beta,\]  
\[(A6)\]
where $\beta = \frac{c}{2} \Delta T$ is small and equal to $1.2 \times 10^{-3}$ for $c = 7.5 \times 10^{-6} \text{ sec}^{-1}$ and $\Delta T \sim 31.04$ sec used in our calculation.

Using the computational scheme shown in Fig. 2, it has been found (Miyazaki, 1965) that, for semi-implicit formulation of friction terms,

$$\lambda_1 = 1 - \frac{\beta}{1 + \beta} \sim 1 - 2\beta, \quad \lambda_{2, a} = \frac{1}{1 + \beta} \left( \phi - \frac{\phi^3}{4} - \beta^2 \right)^{\frac{1}{2}}$$

(A7)

where $\phi = gD \left( \frac{\Delta T}{\Delta s} \right)^2 \left( \sin^2 k\Delta s + \sin^2 \lambda \Delta s \right)$. Applying the same method, $\lambda^*$ can be calculated as

$$\lambda_1^* = 1 - 2\beta, \quad \lambda_{2, a}^* = 1 - \beta - \frac{\phi}{2} \pm i \left( \phi (1 - \beta) - \frac{\phi^3}{4} - \beta^2 \right)^{\frac{1}{2}}$$

(A8)

Comparing (A6), (A7) and (A8), the moduli of $\lambda^o$, $\lambda^*$ and $\lambda^i$ become

$$|\lambda_1^o| = |\lambda_1^i| = |\lambda_2^o| = 1 - 2\beta$$

$$|\lambda_{2, a}^o| = |\lambda_{2, a}^i| = |\lambda_{2, a}^*| = 1 - \beta$$

Thus, the damping rate of the system for explicit and semi-implicit formulation of friction terms is the same and equal to the analytical value.

Using the definition of $\lambda$, we can obtain the oscillation frequency in explicit and semi-implicit cases from

$$\tan \omega^* \Delta T = \pm \left( \phi(1 - \beta) - \frac{\phi^3}{4} - \beta^2 \right)^{\frac{1}{2}}$$

(A9)

and

$$\tan \omega^i \Delta T = \pm \left( \phi - \beta^2 - \frac{\phi^3}{4} \right)^{\frac{1}{2}}$$

(A10)

For the long waves considered in this paper, $\omega$ and $k$ are small, and $\phi \sim gD(\Delta T)^2 k^2$, equations (A9) and (A10) becomes

$$\omega^* \sim \pm \left( gDk^2 - \frac{c^3}{4} \right)^{\frac{1}{2}} \left( 1 + \beta + \frac{\phi}{2} \right)$$

and

$$\omega^i \sim \pm \left( gDk^2 - \frac{c^3}{4} \right)^{\frac{1}{2}} \left( 1 + \frac{\phi}{2} \right)$$

For typical values of $\omega \sim 10^{-4} \text{ sec}^{-1}$ (such as $M_2$, $M_4$) and $|k| \sim 10^{-5} \text{ cm}^{-1}$ estimated from (A4), $\phi$ is $\sim 10^{-5}$. To get the same dispersion relation as given in (A4), $\omega^i$ has to be reduced by $\sim 10^{-3} \%$ and $\omega^*$ by $\sim 10^{-2} \%$. These frequency shifts are small and can be ignored.

REFERENCES


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