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A note on estuarine mean flow estimation

by Robert H. Weisberg

ABSTRACT

The sampling interval required for estuarine mean flow estimation is examined. Examples drawn from the literature show that nontidal estuarine flow is highly variable and that under common everyday conditions this variance is predominantly wind-induced. Thus, record lengths sufficiently longer than the time scale of locally energetic wind fluctuations are required. A nondimensional parameter expressing the number of tidal cycles which must be averaged over to attain a given error tolerance is offered. Since this parameter may be estimated prior to a measurement program it may serve as an aid in estuarine experimental design.

1. Introduction

Mean value estimation is generally a foremost concern in the measurement and description of any quantity. The mean distributions of velocity, temperature, salinity, nutrients, dissolved oxygen, etc. are required for assessing such things as transports, residence times, biological potential, and quality of estuarine waters. The estuarine literature contains many estimates of mean properties acquired by averaging over a few tidal cycles. An appropriate question to ask is what significance do mean property estimates have with respect to the true mean value, or more specifically: how long must an estuarine sample be so that its mean value is representative of the steady state?

The quantity to be treated in this paper is the velocity since the distribution of other estuarine properties ultimately depends to a large extent upon advection. Examples of mean axial velocity components (the horizontal velocity component directed along the longitudinal axis of the estuary) will be drawn from the literature in order to show the intrinsic variability of nontidal estuarine flow. The effects of this variability upon mean value estimation will then be analyzed with specific application to a 51 day current meter record from the Providence River, Rhode Island. Finally, the averaging interval required to obtain a given error tolerance for the mean estimate will be offered as a nondimensional parameter.

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2. Background

Pickard and Rodgers (1959) reported a wind-induced shifting of the mean velocity profile in the Knight Inlet, British Columbia (a fjord-type estuary) based upon averages formed from overlapping 25 hour samples. Similar behavior was observed in the West Passage of Narragansett Bay by Weisberg and Sturges (1976). They found that the sense of the mean axial velocity profile averaged over 39 days was not statistically significant, and it was concluded that the gravitationally convected portion of the flow in the West Passage could not be extracted from current observations irrespective of record length. Fig. 1 shows the estimated mean and the nontidal variability of their measurements. Within one standard deviation of the nontidal current time series about its mean value, the flow was observed to be either landward or seaward over the entire water column in response to the wind.

Weisberg (1975) analyzed the effects of wind and other atmospheric inputs upon the flow in the Providence River using a 51 day velocity record sampled 2 m from the bottom. The observed axial component of velocity and a low pass filtered version of it (excluding semidiurnal tidal and higher frequency oscillations) are shown in Fig. 2. Although a steady mean value is evident, of more interest are the large nontidal fluctuations comprising 48% of the total variance. These fluc-
tuations were almost exclusively wind-induced with 97% of the nontidal axial current variance being coherent with the maximum fetch wind velocity component. Owing to this extremely high coherence, a linear time invariant stochastic model reproduced the axial current from the two orthogonal wind velocity components to within an r.m.s. error of 2.3 cm/s. This result is shown in Fig. 3. It was concluded that the effects of wind can permeate the entire water column of a partially mixed estuary, and can comprise a portion of the total circulation equal to, if not larger than, that of gravitational convection or the tides. Since the Providence River is typical of a coastal plain, partially mixed, estuary this data will serve as a working example for the analysis to follow.
Table 1. A partial listing of replicate mean speed estimates reported in the estuarine literature. Upper refers to the mean speed observed near the surface. Lower refers to the mean speed observed within a distance of 20% of the total depth from the bottom. Positive denotes seaward flow and negative denotes landward flow. Values have been rounded off to the nearest cm/sec.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Upper cm/s</th>
<th>Lower cm/s</th>
<th>Sampling Interval</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pritchard (1967)</td>
<td>+13</td>
<td>-9</td>
<td>18-23 June 1950</td>
<td>James River</td>
</tr>
<tr>
<td></td>
<td>+13</td>
<td>-13</td>
<td>26 June-7 July 1950</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+14</td>
<td>-12</td>
<td>17-21 July 1950</td>
<td></td>
</tr>
<tr>
<td>Dyer (1973)</td>
<td>+8</td>
<td>-6</td>
<td>6 April 1966</td>
<td>Southampton Water</td>
</tr>
<tr>
<td></td>
<td>+7</td>
<td>+8</td>
<td>5-6 May 1966</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+7</td>
<td>0</td>
<td>24-25 May 1966</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2</td>
<td>+12</td>
<td>9 August 1966</td>
<td></td>
</tr>
<tr>
<td>Bowden (1963)</td>
<td>+13</td>
<td>-5</td>
<td>30 June-2 July 1959</td>
<td>Mersey Estuary</td>
</tr>
<tr>
<td></td>
<td>+18</td>
<td>-10</td>
<td>28-29 July 1960</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+17</td>
<td>-9</td>
<td>6-8 July 1959</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+18</td>
<td>-4</td>
<td>1 August 1960</td>
<td></td>
</tr>
<tr>
<td>Bowden and Sharaf el Din (1966)</td>
<td>+2</td>
<td>-13</td>
<td>25 Hr. Samples</td>
<td>Mersey Narrows</td>
</tr>
<tr>
<td></td>
<td>+6</td>
<td>-4</td>
<td>During July 1962</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+28</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+11</td>
<td>-4</td>
<td>27 January 1967</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+9</td>
<td>-5</td>
<td>9 February 1967</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+2</td>
<td>-4</td>
<td>15 February 1967</td>
<td></td>
</tr>
</tbody>
</table>

Replicate mean speed estimates from the literature provide further examples of estuarine nontidal flow variability. Some of these are presented in Table 1. Included are the maximum observed near-surface speed (upper), the speed obtained within 20 percent of the total depth from the bottom (lower), the sampling interval, and the data source. With the exception of the Vellar estuary, which approaches a salt wedge, all of the estuaries included in the table are partially mixed. As would be expected the minimum replicate variability shown in the table is associated with Pritchard's (1967) data, since his record lengths were the longest (up to 11 days in duration). All of the others which have averaging intervals of only one or more tidal cycles show considerable variability, including flow reversals. In view of the large fluctuations associated with nontidal estuarine flow, the rule of thumb of averaging over just a few tidal cycles, as often suggested in the literature, may be a serious mistake.
3. Analysis

A mean value estimate \( \hat{u} \) derived from a time series \( u(t) \) is

\[
\hat{u} = \frac{1}{T} \int_{0}^{T} u(t) \, dt ,
\]

where \( T \) is the averaging interval. If the length of the averaging interval is fixed, but its position relative to the entire time history of \( u(t) \) is shifted, then a different value of \( \hat{u} \) will result. Therefore, the mean estimate \( \hat{u} \) has a variance, denoted by \( \text{var} \, \hat{u} \), which also depends upon the averaging interval. Knowledge of \( \hat{u} \), \( \text{var} \, \hat{u} \), and the probability distribution function for \( \hat{u} \) enables an investigator to assign a confidence interval to the mean value estimate. An averaging interval, or record length, may then be chosen to make this confidence interval suitably small.

Two approaches may be used to calculate the variance of the mean estimate for an arbitrary random variable. The first expresses the variance of the mean in terms of the auto-covariance function \( c(\xi) \) as:

\[
\text{var} \, \hat{u} = \frac{1}{T} \int_{-\infty}^{\infty} c(\xi) \, d\xi ,
\]

where \( T \) is the averaging interval and \( \xi \) is the correlation lag (for a derivation see Papoulis, 1965, p. 323-325, and for an application to oceanographic data see Sturges, 1974). Equation 2 may be used provided that the maximum lag is small compared to \( T \) and that the auto-covariance function is absolutely integrable; i.e., purely periodic (deterministic) fluctuations must be filtered out. Since the spectral density function \( G(f) \) is the Fourier transform of \( c(\xi) \), i.e.,

\[
G(f) = \int_{-\infty}^{\infty} c(\xi) e^{-2\pi if\xi} d\xi ,
\]

the integral on the right-hand side of equation 2 equals \( G(0) \). Therefore the variance of the mean estimate may be obtained directly from the spectral estimate at zero frequency and the averaging interval, or

\[
\text{var} \, \hat{u} = \frac{G(0)}{T} ,
\]

Conceptually, \( G(0) \) represents the portion of signal variance occurring at time scales longer than \( T \), along with contributions from the side lobes of the frequency response function for a uniform average. \( G(0) \) may be estimated, in the case of stationary data, by averaging over the lowest fundamental frequency bands for a desired number of degrees of freedom or stability.

Blackman and Tukey (1958) offer a second approach for computing the variance of the mean estimate by calculating the effective bandwidth of the random variable's spectral density function. If \( \Psi^2 \) is the total signal variance, then \( \text{var} \, \hat{u} \) for bandwidth limited white noise is
where $B$ is the bandwidth and $2BT$ is the number of degrees of freedom. If the spectrum is not white (uniform), then an equivalent bandwidth $B_e$ and an equivalent number of degrees of freedom $n$ may be defined as:

$$B_e = \left[ \frac{\int_0^\infty G(f)df}{\int_0^\infty G^2(f)df} \right]^2$$

and

$$n = 2B_eT .$$

The variance of the mean estimate then becomes:

$$\text{var} \hat{u} = \frac{\psi^2}{n} .$$

Table 2. Two separate approaches to the calculation of the standard deviation of the mean value estimate ($\text{var} \hat{u}$).

<table>
<thead>
<tr>
<th>$G(o)$</th>
<th>$T$</th>
<th>$B_e$</th>
<th>$\psi^2$</th>
<th>$[G(o)/T]^{1/2}$</th>
<th>$[\psi^2/2B_eT]^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm$^2$/s$^2$/c.p.h.</td>
<td>hrs.</td>
<td>c.p.h.</td>
<td>cm$^2$/s$^2$</td>
<td>cm/s</td>
<td>cm/s</td>
</tr>
<tr>
<td>990</td>
<td>1227</td>
<td>0.03</td>
<td>94</td>
<td>0.9</td>
<td>1.1</td>
</tr>
</tbody>
</table>

These two developments for the variance of the mean estimate were applied to Weisberg's (1975) data and the results are summarized in Table 2. $G(o)$ was obtained by averaging the first 12 fundamental frequency bands (approximately 24 degrees of freedom). Since this average included the most energetic portion of the spectrum (4-5 day periodicities) it is the maximum estimate obtainable for $G(o)$ from the given data. $B_e$ and $\psi^2$ are respectively the effective spectral bandwidth and the variance of the axial current excluding the deterministic $M_2$ tide, and $T$ is the record length. The last two columns show the positive square roots of the mean estimate variances, or the standard deviations of the mean.

Given the standard deviations, confidence limits may now be placed upon the mean estimate. The Central Limit Theorem (e.g., Bendat and Piersol, 1971, p. 111) states that the probability distribution function for the mean of a random variable very rapidly approaches a Gaussian distribution irrespective of the distribution function of the variable (which was approximately Gaussian in the present case). Assuming this, the 80% confidence interval for the mean value becomes:

$$\hat{u} = 11.7 \pm 1.4 \text{ cm/s} ,$$

Therefore, if the experiment were to be repeated and the velocity statistics remained stationary then the resulting mean value would be expected to fall within the given interval with an 80% certainty.
The calculation procedure may now be inverted to obtain the record length required for a given error tolerance. Since mean values vary spatially within an estuary, it is helpful to consider a normalized error $\epsilon$ (Bendat and Piersol, 1971, p. 172),

$$\epsilon = \frac{\text{var} \ \hat{u}}{\bar{u}} .$$

(9)

Using the Blackman and Tukey (1958) formulation, the record length with respect to the normalized error becomes:

$$T = \frac{\Psi^2}{2B_e \epsilon^2 \hat{u}^2} .$$

(10)

Equation 10 may be nondimensionalized by dividing both sides by $T'$, the semi-diurnal tidal period. The resulting parameter $\tau$ expresses the number of tidal cycles that must be averaged over to obtain a normalized error of less than $\epsilon$:

$$\tau = \frac{\Psi^2}{2B_e T' \epsilon^2 \hat{u}^2} .$$

(11)

The facility of $\tau$ is that it depends upon quantities which may be estimated in advance of measurements. Both $\Psi^2$ and $B_e$ are primarily due to local winds and $\hat{u}$ may be anticipated from previous estuarine studies or from conservation considerations for an idealized two-layered mean flow. For example: a typical wind speed of 5 m/s and a wind factor (ratio of wind speed to water speed) of about 0.02 (Wu, 1973) would result in a $\Psi^2$ of $10^3 \text{cm}^2/\text{sec}^2$. The bandwidth over which this variance is spread is determined by the bandwidth of the energetic wind fluctuations. With the exclusion of the summer sea breeze a conservative upper limit for $B_e$ is about 0.03 c.p.h. (Table 2). If a mean value of about 10 cm/s is expected and a tolerance of 0.2 is specified then the required record length is about 36 tidal cycles or 18 days.

Thus, even during common everyday conditions, estuarine record length requirements are large. This result is particularly important in view of the fact that the most recent estuarine text (Dyer, 1973) still suggests that a suitable averaging interval is a couple of tidal cycles. Putting the previous statistical discussion aside, a glance at Fig. 2 clearly demonstrates that averaging over only a few tidal cycles results in mean value estimates ranging anywhere between −4 and +25 cm/s which seems meaningless for most physical or water quality applications.

4. Summary

Data from previous estuarine studies have been recalled demonstrating the intrinsic variability of nontidal estuarine flow which under common everyday conditions is predominantly wind-induced. With this as motivation, the record length requirement for mean flow estimation in an estuary has been examined. A non-
dimensional parameter $\tau$ has been offered expressing the number of tidal cycles that must be averaged over to obtain a given error tolerance:

$$\tau = \frac{\Psi^2}{2B_e T' \epsilon^2 \bar{u}^4} ,$$

where the quantities are defined in the text.

Tacitly assumed in the analysis was stationarity. Since energetic wind fluctuations occur on the time scale of a few days, record lengths must be at least that long before meaningful statistics can be computed. How much longer depends upon the random error tolerance that one is willing to accept. This paper has emphasized wind-induced random errors occurring during common everyday conditions. Large storm events (with accompanying atmospheric pressure and river runoff effects) and systematic seasonal changes result in further nontidal variability of a nonstationary nature which should be treated separately.

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REFERENCES


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