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A Numerical Experiment for the Path of the Kuroshio

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ABSTRACT

There are two stable patterns of flow for the Kuroshio south of Japan: (i) flow along the continental slope, all the way east to the Izu-Ogasawara Ridge; and (ii) a southeastward meander away from the continental slope south of Shikoku, out over deep water. We have modeled the Kuroshio as a steady free inertial jet that extends to the bottom over at least a portion of its path. An analysis of solutions to the path equation has been made; two mechanisms for the production of a bimodal distribution of the paths are proposed. If there is a change from a large bottom velocity (strong topographic control) to a zero bottom velocity (inertial-Rossby wave), the calculations with the model predict that the Kuroshio will switch from a path over the slope to a meander path. There is strong evidence that the Kuroshio meander over the Shikoku Basin may be interpreted as an inertial-Rossby wave. A more subtle mechanism for the production of bimodal paths allows the Kuroshio to maintain a bottom velocity over a portion of its path; but, at a turn-off depth (about 3000 m), the bottom interaction weakens and the current effectively becomes an inertial-Rossby wave, unless it encounters shallow water again. The existence of a turnoff depth produces a shadow zone, where solutions for paths that are run over the actual topography only rarely penetrate. The position of the shadow zone for the solutions is qualitatively similar to the zone between the two characteristic path modes. Solutions outside the shadow zone fit the envelopes of observed Kuroshio paths.

1. Description of Kuroshio Path and Statement of Model.

1.1. Introduction. The Kuroshio is the northward-flowing western-boundary current of the North Pacific. Hydrographic measurements in the Kuroshio that date back more than 40 years have revealed striking shifts in the position of the

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current south of Japan (Uda 1964): the Kuroshio, for long periods of time, is located over the upper portion of the continental slope; however, there is another stable pattern of flow, when the Kuroshio leaves the continental slope and flows southeastward out over the deep offshore water south of Honshu. This offshore displacement has been referred to in a variety of ways, often as the Kuroshio meander. These changes in the current are characterized by their long-term stability (both patterns of flow having life times of at least several years) and by their almost bimodal character. Taft (in press) has analyzed the available data from the Kuroshio for the period 1956 through 1964. During this period the Kuroshio has shifted its position from one quasistationary pattern to the other. An analysis of these data has led to a description of the Kuroshio flow characteristics that are associated with each of the two patterns.

Robinson and Niiler (1967) have presented a general theory of free inertial currents that has been applied to a study of the Gulf Stream meanders (Niiler and Robinson 1967) and to a study of the East Australian Current (Godfrey and Robinson 1971); in this paper the theory is applied to the dynamics of the Kuroshio south of Japan. The vorticity equation, which, in a steady state determines the path of a free inertial current, contains inertial, advective, topographic, and planetary contributions to the vorticity balance; their relative importance is measured by integral properties of the current profiles. A variety of qualitatively distinct flows is possible. Solutions to the path equation are obtained by numerical integration over the particular terrain south of Japan, for the broad range of current parameters provided by an analysis of available hydrographic data. These solutions constitute the steady-state paths available to this model current. The distribution of the solutions obtained is bimodal; the underlying mechanism that produces this result is described and proffered as a possible explanation for the bimodal behavior of the Kuroshio. The approach to the analysis of the phenomena adopted here, via the study of local dynamics, complements any approach to an explanation of the phenomena by a consideration of larger-scale processes, e.g., climatological process.

1.2. Bottom Topography South of Japan. South of Japan, the northern end of the Shikoku Basin is bounded by the steep continental slope on the west and north and by the Izu-Ogasawara Ridge on the east (Fig. 1). The 4000-m isobath roughly marks the base of the continental slope and may be used as the boundary of the Basin. The floor of the Basin is generally rather flat. There are three major seamounts in the Basin; the largest is at 31°30'N, 136°E.

Generally, the continental slope runs parallel to the coastlines of the islands. The trend of the slope changes from northeast off Kyushu to east-by-north south of the Kii Peninsula. Except for a short broad section of the slope seaward of the channel between Kyushu and Shikoku, the slope in this region
generally steepens offshore. There are two regions where the slope abruptly changes both orientation and width: near 133°E, the trend changes from north-east to northeast-by-east and the width of the slope decreases by one half; near 134°30′E, the trend changes from northeast-by-east to east-by-north and the width of the slope again decreases by about one-half. The narrowest section of the slope is located between 134°30′E and 135°E (n.b., this feature is important in the ensuing arguments). East of the Kii Peninsula the deeper portion of the slope does not follow the coastline. The coastline turns to the north, but the east-by-north trend of the 2000-, 3000-, and 4000-m isobaths does not change; there is a plateau on the upper portion of the slope.

The continental slope meets the western edge of the Izu-Ogasawara Ridge at about 138°E. The Ridge deepens gradually to the south, from depths of less than 500 m at its northern end to over 2000 m at 30°N. Unless the Kuroshio does not extend to the bottom, the northern section of the Ridge presents a barrier to the flow of the deep water underneath the Kuroshio. However, in this paper we are concerned with an analysis of the behavior of the Kuroshio before it reaches the Ridge.
1.3. Path of the Kuroshio. Observations. Systematic surveys of the offshore waters south of Japan have been made since 1955. Direct measurements of the surface-current velocity with the GEK (geomagnetic electro-kinetograph) were made on transects orientated normal to the coastlines. The interval between the velocity measurements on these transects was between 20 km and 35 km. Most of these transects show a single well-defined maximum of velocity that can be identified as the axis of the Kuroshio at the sea surface (Taft, in press). There is a consistent relationship between the location of the maximum surface-current speed and the subsurface temperature distribution (Uda 1964, Kawai 1969). The maximum GEK surface-current speed almost always lies above the strongest horizontal temperature gradient that occurs at a depth of 200 m. In an analysis of bathythermograph temperature measurements south of Japan, Kawai has shown that the strongest horizontal temperature gradient at 200 m occurs at a temperature close to 15°C, so the axis of the Kuroshio at the surface can alternatively be depicted by plotting the position of the 15°C isotherm at 200 m. This type of identification of the position of the axis of the Gulf Stream near the sea surface was introduced by Fuglister (1963) and is commonly and usefully employed (e.g., vid. Hansen 1970). In the Kuroshio south of Japan, it appears that there would be no significant difference between a representation of the current axis based on GEK surface-current measurements and a representation based on the temperature at 200 m. It should be pointed out that this representation applies to the left-hand side of the current. The surface-current distribution is asymmetrical, so that the maximum speed is much closer to the inshore side of the Kuroshio (Uda 1964).

Taft (in press) has presented 43 charts that show the position of the maximum surface-current speed of the Kuroshio. The curves that connect the speed maxima on the charts are taken to represent the path of the Kuroshio. One of these charts is reproduced in Fig. 2. From August 4 to 26, 1956, 14 GEK transects were made across the Kuroshio; the spacing between the transects varied from 50 km to 100 km. The transects were repeated at both 135°40'E and 137°E. The picture is not synoptic because of the large distance between the transects and because of the 22-day time interval between the beginning and the end of the survey. Fuglister (1963) has pointed out emphatically that the curvature in the path of the Gulf Stream cannot be inferred from widely spaced north-south transects across the current. In this paper, the heavy line denoting the path is used only to show the position of the strongest surface current. It is understood that the interpolated line between the transects has little significance.

The August 1956 GEK measurements indicate that the Kuroshio west of the Kii Peninsula was located above the upper portion of the continental slope at a depth between 1000 m and 1500 m. East of Cape Shionomisaki, the southernmost point of the Kii Peninsula, the Kuroshio apparently flowed in
Figure 2. Surface-current measurements made by GEK during August 4 to 26, 1956. The location of the maximum surface current on each transect is designated with a black triangle, which is orientated so that the apex of the triangle points in the direction of the maximum surface current. The number adjacent to the triangle indicates the maximum speed in knots. The black dots on the solid transect lines indicate speeds that are greater than 0.9 knots with a direction roughly parallel to the flow at the Kuroshio axis. Dashed portions of the transect lines indicate regions where the speeds are less than 1.0 knot and appear to be outside the strong flow of the Kuroshio. On the dashed section of the transects, where the speeds are in excess of 0.9 knots, the current vectors are given. The conversion scale for the vectors is in the lower left-hand corner.

Note that the axis of maximum geostrophic current shear slants offshore with increasing depth so that the representations in Figs. 2 and 3 probably are not valid if we consider the position of maximum speed of the Kuroshio...
beneath the sea surface. Masuzawa and Nakaii (1955) have indicated that the maximum current at a depth of 500 m, relative to 1000 m, may be displaced offshore 35 km to 70 km from the position of the maximum surface-current speed. The offshore displacement of maximum current is taken into account in the mathematical model used in the numerical experiment.

**Slope and Meander Paths.** The GEK observations strongly suggest that the path data are bimodal and that the current east of Shionomisaki is either above the slope or is meandering over the Basin. If all of the current paths for 1956 through 1964 were plotted on the same chart, it would be clear that, during this nine-year period, almost all of the paths would resemble either the path that remains above the continental slope (Fig. 2) or the path that meanders (Fig. 3). In order to simplify the discussion, a path that remains above the continental slope until the current passes Shionomisaki will be referred to as a continental slope path, or *slope path*. The type of path that leaves the continental slope and turns southeastward east of Shikoku will be referred to as a *meander path*. Figs. 4 and 5, respectively, show composite pictures of all the available slope and meander paths during the 1956-through-1964 period.
After the meander was set up in May 1959, the current continued to flow in the meander pattern for over three years (Shoji 1965; Taft, in press). All of the evidence from the surveys indicates that the current did not return to the continental slope off Shionomisaki until May 1963. During March 1963 the current was in an intermediate position between the slope path and the meander path. It appears that the meander was set up in less than three months and disappeared during a period of about six months. Observations were not made during each month, so it is possible that an undetected northward movement of the current occurred during the 1959-to-1962 period, but this is unlikely. The data indicate that the current meander was stable for 3 to 4 years.

The large-scale stable meander of the Kuroshio has been known to Japanese oceanographers for a long time. Uda (1937, 1940) gave the first scientific description of the meander. More recent discussions of the meander are found in Moriyasu (1963), Uda (1964), Shoji (1965), Masuzawa (1965), and Yoshida (in press). During the 40 years when oceanographic data have been collected off Japan, the meander has been observed three times: 1934 to 1944, 1953 to 1955, and 1959 to 1963. Since systematic oceanographic coverage of the area south of Japan commenced in 1955, the last occurrence of the meander is the best documented. For this reason our discussion of the meander is based on the 1959-through-1963 observations.

Observations in the Atlantic between the Straits of Florida and Cape Hatteras do not show a long-period stable offshore displacement of the Gulf Stream from the continental slope (Stommel 1965). However, it may be a more appropriate analogy to regard the Gulf Stream leaving the coast at Cape Hatteras as a truly permanent large-scale meander, which is complicated to the east by smaller-scale slow but transient meanderings that are commonly referred to as the Gulf Stream meanders. The available understanding of these phenomena, both experimental and theoretical, has been reviewed by Robinson (1971). The processes associated with the movement of the Gulf Stream away from the Cape are not understood; they may be complicated by an interaction of the Gulf Stream with the deep flow along the boundary (Richardson and Knauss 1970).

Theoretical Approaches. Ichiye (1955, 1956) applied the barotropic shear instability theory of Haurwitz and Panofsky (1950) to the Kuroshio and concluded that the meander might be considered as an eastward propagating instability in the current. The computed phase speed of the wave was about 0.5 km. Since the meander is quasistationary, the phase speed computed by Ichiye is much too high, and the instability theory does not appear to account for the phenomenon.

Some writers have considered the effects of topography on the flow of the Kuroshio south of Japan and have speculated on their possible role in producing the meander of the Kuroshio (Hayami et al. 1955, Fukuoka 1957, 1960,
Ichiye 1960). These studies were based on highly idealized models of the effects of barriers on the flow of a current (Bolin 1950, Kajiura 1953), and as such they are not directly applicable to the Kuroshio.

Other authors have examined a correlation between the occurrence of the meander and the changes in the atmospheric or oceanic circulation in the western Pacific (Uda 1949, Nan' niti 1958, Masuzawa 1965, Namias 1970, Taft, in press). It is possible that the meander is physically associated with large-scale changes in the circulation of the western Pacific, and insight into the dynamics of the meander eventually may come from correlating its advent with changes outside the Kuroshio region. However, the effect of these changes in the large-scale circulation on the local dynamics of the Kuroshio meander has not been explored.

In this paper the mathematical model of a free inertial current is used to compute the path of the Kuroshio south of Japan. In particular, we explore the hypothesis that the Kuroshio south of Japan is steered by a novel topographic mechanism of control that results in a strong tendency for the Kuroshio to assume one of the two characteristic paths. A corollary of this hypothesis is that paths intermediate between the two-path envelopes shown in Figs. 4 and 5 probably are not steady-state positions of the current.
1.4. **The Mathematical Model.** The path equation, which determines the location of the axis of a steady free inertial current in terms of integral properties of the current profile, has been formally derived from the Boussinesq ideal fluid equations within the framework of a general theory (Robinson and Niiler 1967; § 5). A canonical form of the equation, for application in numerical experiments on the paths of real ocean currents, has been presented by Niiler and Robinson (1967; § 2). Here we merely state the model in the special form used and summarize the most important assumptions that govern its applicability to the computation of Kuroshio paths.

Let \((X, Y, z)\) be positive, respectively, in the eastward, northward, and upward directions; and \((x, y)\), in the cross-stream and downstream directions in a local curvilinear coordinate system, is defined so that the origin of the \(x\)-coordinate coincides with the axis of the current, \(Y = Y(X)\) (Fig. 6). The curvature of the axis is given by

\[
\kappa = \frac{d\theta}{dy} = \frac{d^2 Y}{dX^2} \left[ 1 + \left( \frac{dY}{dX} \right)^2 \right]^{-3/2}.
\]

The cross-stream and downstream velocity components are denoted by \((u, v)\), and the sea bottom, sea surface, and Coriolis parameter are denoted by \(B, H,\)
and $f$, respectively. The Coriolis parameter ($f$) is $2\Omega \sin \varphi$, where $\Omega$ is the angular speed of the earth’s rotation and $\varphi$ is the latitude. In the $x,y,z$ coordinate system, the cross-stream $x$-momentum equation takes the approximate form

$$-\kappa u^2 - f v + \frac{1}{\rho} p_x = 0, \tag{2}$$

where $\rho$ is the density and $p_x$ is the cross-stream pressure gradient; the downstream $y$-momentum equation is exact (Robinson and Niiler 1967). If the equation for the vertical component of vorticity is integrated in depth and across the current, the result is

$$\int dx \left\{ \int_B^H dz \left[ \frac{\partial}{\partial y} (\kappa u^2) + v \frac{\partial f}{\partial y} + f v (B) \frac{\partial B}{\partial y} \right] + \int_B^H dz (\nabla \cdot \mathbf{v} + \kappa u v) \right\} \bigg|_{\text{edges}} = 0. \tag{3}$$

Mass continuity and the bottom-boundary condition of vanishing normal velocity have been used in the derivation of (3).

The model is based on the following assumptions:

(i) The current is free. Thus the mass and sea-level distributions adjust on both sides of the current to give pressure gradients in the open ocean that are much weaker than the gradient across the jet that supports the strong axial flow; there is no wall pressure. In particular, the last term of (3), the edge-terms, vanishes.

(ii) The current is narrow. The curvature, the Coriolis parameter, and the depth are taken constant across the current:

$$\kappa \propto \kappa(y) \text{ only}; \quad f = f_0 + \beta Y \propto f(y); \quad B(X,Y) \propto B(y).$$
(iii) The radius of curvature of the meanders is larger than the width of the current.

(iv) The depth changes are small compared with the total depth. The vertical integral in (3) is taken over the average depth, for which \( B \equiv 0 \) by definition.

(v) Downstream changes in both the vertical and cross-stream profiles of the current are negligible over the path segment of interest, i.e., \( v(x, y, z) = v(x, z) \) only.

Assumptions (i), (ii), (iii) are implicit in the derivation of the integro-differential eq. (3). Under (iv), one integration in \( y \) is possible; and under (v), the resultant equation becomes an ordinary differential equation for \( Y(X) \) when expressed in geographical \( X, Y \) coordinates with the use of (1). The result is

\[
\frac{d^2 Y}{dX^2} \left[ 1 + \left( \frac{dY}{dX} \right)^2 \right]^{-3/2} - C_0 + C_1 (B - B_0) + C_2 (Y - Y_0) = 0, \tag{4}
\]

where a zero subscript refers to the point \((X_0, Y_0)\), and

\[
C_0 \equiv 10^6 \chi_0, \quad C_1 \equiv \frac{10^{10} \bar{\nabla}}{\langle v^2 \rangle}, \quad C_2 \equiv \frac{10^{12} \beta <v>}{\langle v^2 \rangle}. \tag{5}
\]

The bracket notation denotes double integration across the current profile and over the average depth; the overbar expresses integration across the current along the average bottom. In the following text, units will not be stated for the \( C \)'s; it is understood that they are \( 1 \) cm\(^{-1} \) for \( C_0 \) and \( 1 \) cm\(^{-2} \) for both \( C_1 \) and \( C_2 \).

If the requisite integral moments of the current are known, then eq. (4), which contains the effects of inertia \((C_0)\), of bottom-topographic vortex-line stretching \((C_1)\), and of planetary vorticity tendency or \( \beta \) effect \((C_2)\), determines the path \( Y(X) \) of the current. The path is obtained through the solution of a spatial initial-value problem by integrating forward in \( X \) from some initial point \((X_0, Y_0)\). The initial current direction \([(dY/dX)_0 , \theta_0] \) and the initial curvature \((C_0)\) are also required. The equation describes a variety of qualitative phenomena that arise because of, and are characterized by, the nature of the underlying terrain described by \( B(X, Y) \). The region of interest for Kuroshio-path computation described in § 1.2 has been digitalized in a set of four sub-tables. The method of numerical integration has been described by Noller and Robinson (1967; § 1.3); the maximum step size used in this study for the path calculations was 15.66 km.

In its course around the southern edge of Kyushu over to the Izu-Ogasawara Ridge (Figs. 1–5), the Kuroshio may violate assumptions (i) free flow or (iv) small depth changes of the model, either by approaching close to the shore or by flowing off the continental slope into deep water. Thus the model pos-
tulated here is not applicable to the current over an entire length of path in this region. This difficulty will be overcome by performing path integrations over shorter path segments, thereby elucidating the local dynamical behavior. The longer path lines will then be discussed by joining together segmental integrations via plausible physical arguments. This approach obviates the necessity of a more sophisticated description of the profiles than that assumed in (v). We believe that the simple model investigated here contains many processes pertinent to the flow of the Kuroshio and that the study of this model provides a logical first step in the development of a theoretical understanding of the meander phenomenon.

2. Behavior of Path Solutions over Bottom Topography South of Japan.

In this section we show a series of solutions that exhibit the various types of paths that may be computed with the model over the terrain of interest. The properties of the computed paths are used to establish the proper range of the parameters defined in (4). These parameters are for the calculation of current paths to be used subsequently in numerical experiments that will be compared with observed paths.

2.1. Type of Bottom Control of Current. The different paths obtained by varying the extent of the topographic control can be shown by setting $C_0 = C_2 = 0$ in (4) and thence varying $C_1$. The control curve for the path, i.e., the curve connecting points of zero curvature, is an isobath. The current meanders about the isobath; the amplitude of the meandering is determined by the relationship between the (local) wavelength of the pure topographic wave ($C_2 = 0$) and the lengthscale characteristic of the variations in water depth. It can be shown from (4) that the wavelength of small-amplitude oscillations about the control curve for a current flowing over a uniformly sloping bottom is $\pi(C_1s)^{-1/2}$, where $s$ is the slope of the bottom topography. A discussion of the behavior of a current flowing over a uniformly sloping plane bottom has been presented (Robinson and Niiler 1967; § 3); these solutions may be helpful in the interpretation of the following results. The behavior over a flat bottom but with variable Coriolis parameter is completely analogous; a latitudinal circle plays the role of an isobath.

In Figs. 7 and 8, calculated current paths are shown for a range of $C_1$ values. All of the path calculations are started at the same initial coordinates on the continental slope. These coordinates were chosen so as to fall within the envelopes of the paths shown in Figs. 4 and 5. The depth at the starting point is 1500 m. The solution for $C_1 = 0.001$ is almost a pure inertial path, with little steering by the topography. This solution does not turn eastward as the continental slope changes orientation; it intersects the boundary at the eastern end of Shikoku. As $C_1$ is increased the solutions show more topographic control of the current.
When the wavelength of the pure topographic wave is large relative to the typical length-scale characteristic of the changes in water depth, the flow is controlled by topography in a novel way. The shape of the path is not obviously related visually to the topographic contours, in that the current does not hang on to a depth contour, nor is there a simple wave form about an isobath. But the path solution is gently steered by the bottom topography, and the current meanders about the control curve with a small amplitude. In Fig. 7, the solution for $C_1 = 0.005$ illustrates this type of topographic control of the current; this will be discussed further below. There is an intermediate range of $C_1$, where the wavelength of the topographic wave is comparable to the length-scale of the variations in depth; because of the complex nature of the topography south of Japan, the solutions for this range of $C_1$ are complicated and highly nonlinear. Fig. 8 shows an example in the solution of $C_1 = 0.03$; in this solution the path meanders around the 1500-m isobath with large amplitude. For $C_1 = 0.10$, the amplitude of the meander is less; as the topographic control becomes stronger and as the wavelength shortens, the flow tends to meander more sharply about the relatively slowly changing isobath. These solutions are directed almost normal to the orientation of the isobaths when the current is changing depth. The solution for $C_1 = 1.0$ illustrates the type of solution obtained when the wavelength of the topographic wave is small relative to the
length-scale of the variations in depth. This solution shows small-amplitude meanders about the 1500-m isobath. Topographic control of the current is so tight that, when the depth contours turn sharply, the solution loops across itself at 32°40'N, 133°30'E and thus violates the assumptions of the theory. This type of behavior represents the upper limit of tight topographic control of the current path.

The available evidence favors an interpretation that the Kuroshio is gently steered by the bottom topography. Paths estimated from the data generally do not show large variations in depth over short distances (Taft, in press). In addition, velocity vectors at the current axis do not show large components that are normal to the isobaths. Therefore, the large meanders obtained for intermediate values of \( C_1 \), e.g., 0.03 and 0.10, do not appear to be consistent with the data. Also, computations of \( C_1 \) values from hydrographic sections across the Kuroshio are presented in § 3; it is seen that the bottom velocities that are required to give values of \( C_1 \) in the range where tight topographic control of the current occurs do not seem reasonable. The pure topographic wave of the Kuroshio has a wavelength that is long relative to the length-scale characteristic of changes in the bottom topography in this region. The control that the bottom topography exerts on the current only operates over rather long downstream distances.
2.2. Dependence of Solutions on Type of Bottom Averaging. It has been pointed out that hydrographic sections across the Kuroshio show that the maximum geostrophic current shear at 500 m is generally found seaward of the maximum surface current. For this reason it seems advisable not to use the local bottom depth in running solutions over the topography south of Japan; instead we use an average bottom depth in which more weight is given to the depth to the right of the maximum surface current.

In order to show the influence of the specific choice of form of the bottom term in the equation, pairs of solutions that employ both a local bottom and an averaged bottom have been run for two choices of $C_2$ in the gentle topographic steering range ($C_1 = 0.0045$). The northern solutions in Fig. 9 are pure topographic waves ($C_2 = 0$). The solutions that cross the continental slope and enter deep water (see §2.3 below) at about $133^\circ50'E$ have a $C_2$ of 0.004. The local bottom solutions use the depth underneath the computed path; the other solutions use a two-point average of the local depth and the depth at a distance of 50 km to the right of the path. This averaging scheme has two effects: it generally smooths the bottom topography; it gives greater weight to the depth seaward of the current axis.

The paths plotted in Fig. 9 show that the effect of bottom averaging on
these solutions is not large. For both choices of the parameters the general character of the solutions is not changed. Other calculations with the model suggest that a more complicated averaging scheme is not warranted because of a lack of sensitivity in the solutions to this type of averaging. This insensitivity results from our choice to investigate solutions in the range of gentle topographic steering. In the range of tight topographic control of the current, e.g., when \( C_1 = 0.1 \), there is a pronounced difference in the character of the solutions depending on whether averaged or local bottom topography is used; i.e., the local-bottom solution shows considerably more structure.

All solutions discussed in the remaining portion of this paper are run over a bottom topography averaged 50 km to the right.

### 2.3. Solutions Allowing Displacement of Current from Initial Isobath

The determination of the control curve, which connects the zeros of curvature of solutions, has been discussed for the general case of \( C_1, C_2 \), both finite (Robinson and Niler 1967; § 2.1). In the case where \( C_0 = 0 \), the control curve leaves the initial isobath (where the calculation is begun) at a rate that is a function of the ratio \( C_1/C_2 \). For large values of this ratio, the path tends to meander about a fixed isobath (topographic wave), and for low values of this ratio, the path tends to meander about the initial circle of latitude; i.e., the

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**Figure 10.** Influence of the \( \beta \) effect upon path solutions gently steered by the bottom topography. Solutions were started at the same position as in Figs. 7 and 8.
solution deviates little from a stationary Rossby wave. For values of this ratio near unity, the current paths have neither simple interpretation. For a current started in the northeastward direction on the continental slope off Kyushu, inclusion of the term multiplied by $C_2$ has the effect of moving the solution off the initial isobath into deeper water. In Fig. 10, this effect is seen in the solutions with a fixed $C_1 (0.0045)$ and with different values of $C_2$. The value of $C_1 = 0.0045$ corresponds to a gently steered topographic wave. A decrease in the ratio $C_1/C_2$ brings the solutions down off the upper portion of the continental slope at a more western longitude. The lower the ratio $C_1/C_2$, for a fixed $C_1$, the more rapid the descent of the path into deeper water. This is exemplified by a comparison of solutions: that with $C_2 = 0.002$ reaches a depth of 2000 m at $134^\circ 40' E$ whereas the solution with $C_2 = 0.006$ reaches a depth of 2000 m at $134^\circ 05' E$.

Recall that the solutions are not physically meaningful when the current flows over the large change in depth between the upper portion of the continental slope and the relatively deep water of the Shikoku Basin [Assumption (iv), § 1.3]. In § 5.1, a hypothesis about the flow at the transition between the continental slope and the deep water will be proposed so as to allow physically valid solutions to be continued over the Shikoku Basin. However, it is useful to first discuss paths that are only formal mathematical solutions of (4) when they cross the continental slope.

2.4. Rossby-wave Solutions. Solutions for current paths with zero bottom velocity ($C_1 = 0$) meander about a circle of latitude. Since the topography does not affect the solutions, their forms are independent of starting points. We start, then, with the same initial position and direction as the solutions in Figs. 7 through 10. In Fig. 11 are shown Rossby-wave solutions ($C_0 = C_1 = 0$) for values of $C_2$. As $C_2$ increases, the wavelength of the solution decreases; i.e., the latitude where the solution turns southward moves westward. The wavelength of the Rossby-wave solution for $C_2 = 0.002$ is long; the longitude of the maximum latitude for this solution is $135^\circ E$. The solution for $C_2 = 0.004$ is a rather good fit to a typical meander path over the Shikoku Basin (Fig. 5). The possible significance of the location of the Rossby-wave solution for $C_2 = 0.004$ within the envelope of the meander paths in the deep water of the Shikoku Basin is discussed in § 6. Solutions for values of $C_2$ equal to 0.006 and 0.008 have wavelengths that appear to be significantly shorter than the apparent wavelength of the meander paths.

2.5. Dependence of Solutions on Initial Angle. Solutions to the path equation in the range of gentle topographic steering vary considerably with the current direction at the starting point of the path calculation. To demonstrate this dependence, we hold the starting point fixed and maintain $C_0 = 0$. In Fig. 12 are plotted solutions with initial angles ranging from $30^\circ$ to $75^\circ$. In the solu-
Figure 11. Inertial-Rossby wave-path solutions.

Figure 12. Dependence of path solutions upon initial direction; variations of $\theta_0$, with all other parameters fixed.
2.6. Effect of Changing Starting Point (Zero of Curvature). All of the solutions in Fig. 7 through 12 have been started at the same initial coordinates, with \( C_0 = 0 \). In order to demonstrate the effect of changing the starting point, six solutions with the same \( C_1, C_2 \), and initial angle are plotted in Figs. 13 and 14. This experiment is equivalent to a variation of \( C_0 \) for fixed \((X_0, Y_0)\). Solutions in Fig. 13 are begun at the same latitude (31°N) but with the depth varying from 900 m to 2200 m. The behavior of these three solutions is similar, and the solutions remain parallel to each other out to 136°E. When the initial depth is held constant while the starting point of the solution is moved along the isobath (Fig. 14), the results are very different. There is a crossing of paths at about 135°E. The solution started at 30°30′N tends to remain parallel to the initial isobath whereas the two solutions with more

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Figure 13. Dependence of path solutions upon initial coordinates; variations of \( X_0 \) for fixed \( Y_0 \). All three paths are calculated for the parameters listed in the upper left-hand corner.
northern starting points cross down over the bottom contours with a relatively large angle so that they intersect the southernmost path on the lower part of the continental slope.

Since movement of the point of zero curvature along the continental slope influences the calculated path, it would be desirable to compute from the observational data the location of the zero of path curvature. Unfortunately, this is not possible because of the lack of continuous path data.

3. Parameters in Path Equation Computed from Kuroshio Data. In §2, some illustrative current paths were computed, using values of $C_1$ and $C_2$ that were considered to be in the range of values most relevant to the Kuroshio. In order to compute the values of these two parameters for the Kuroshio, the velocity distribution in both depth and across the current is required. Since the velocity distribution has not been measured directly, magnitudes of the parameters must be estimated by an indirect method. In this section, such calculations are presented and discussed.

3.1. Computed Values. Measurements of the vertical distributions of temperature and salinity across a current can be used to compute the geostrophic component of velocity with respect to an arbitrary reference level. Because the
cross-current balance of forces in the Kuroshio is probably approximately geostrophic, a reliable estimate of the relative current distribution can be obtained in this manner, provided the current is spanned by hydrographic stations and the casts are extended to the bottom. Unfortunately, hydrographic casts on the surveys usually extended to only 1000 m.

In order to use the relative current profiles to compute the values of $C_1$ and $C_2$ (eq. 5), we must make an assumption about the velocity distribution below the reference level. The velocity of the current at the sea bottom, i.e., $u(x, z = B)$, must be known for vertical integrations over depth. Constant values of the bottom velocity across the current are assumed, with the profiles of relative current extrapolated to the sea bottom in the following manner: the layer between 800 m and the sea bottom is assumed to be moving with the constant velocity assumed at the sea bottom. Estimates of the geostrophic vertical shear are used to compute the vertical distribution of the current above 800 m.

Since the hydrographic sections were not always normal to the axis of the Kuroshio, there is a tendency for the geostrophic velocities (but not the total transport) to be underestimated. Since the value of the integral of $v^2$ will be lower when the section crosses the current at an angle other than a right angle, an overestimation of the values of $C_1$ and $C_2$ results. For example, if the section crosses the current at an angle of $45^\circ$, the integral of $v^2$ will be underestimated by a factor of two. In order to minimize this source of error in the calculation of $C_1$ and $C_2$, we have used only sections where the angle between the direction of the maximum GEK surface-current vector and the orientation of the section is between the limits of $75^\circ$ and $105^\circ$.

A lack of knowledge of the bottom velocity prevents accurate estimation of $C_1$ for the Kuroshio; however, the magnitude of $C_2$ is insensitive to the assumed value for the bottom velocity. In Fig. 15 are shown computed values of $C_1$ as a function of the assumed bottom velocity for three representative geostrophic velocity sections. At the northern and southern limits of these sections, the component of flow in the direction of the axial current was less than 10 cm sec$^{-1}$. Thus these hydrographic sections spanned the Kuroshio. The value

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2. This is not inconsistent with eq. (2), since the theory can be alternately expressed quasigeostrophically (Robinson 1971).
Table I. Values of $C_1$ for bottom velocity $v(B) = 1$, $10$ cm sec$^{-1}$, and values of $C_2$ for bottom velocity $v(B) = 1$ cm sec$^{-1}$, computed from selected sections of geostrophic velocity across the Kuroshio during 1956 through 1964.

Depth of water underneath maximum geostrophic surface current is given.

<table>
<thead>
<tr>
<th>Date</th>
<th>Year</th>
<th>Longitude (m)</th>
<th>Depth (m)</th>
<th>$C_1$ cm$^{-2}$</th>
<th>$C_2$ cm$^{-2}$</th>
</tr>
</thead>
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<tr>
<td><strong>MEANDER NOT PRESENT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIII 20-21...</td>
<td>1956</td>
<td>137°40'E</td>
<td>4000</td>
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<td>0.003</td>
</tr>
<tr>
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<td>0.002</td>
</tr>
<tr>
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<td>4000</td>
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<td>0.002</td>
</tr>
<tr>
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<td>0.004</td>
</tr>
<tr>
<td>XI 20-21...</td>
<td>1958</td>
<td>138°00'</td>
<td>4000</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>XI 10-11...</td>
<td>1963</td>
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<td>2500</td>
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<td>0.003</td>
</tr>
<tr>
<td>XI 16-17...</td>
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</tr>
<tr>
<td>V 21...</td>
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<td>2000</td>
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<td>0.003</td>
</tr>
<tr>
<td>VIII 10-11...</td>
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<td>3900</td>
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<td>0.003</td>
</tr>
<tr>
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<tr>
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<td>2100</td>
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</tr>
<tr>
<td>XI 16-17...</td>
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<td>137°00'</td>
<td>2500</td>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td><strong>MEANDER PRESENT</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X 21-22...</td>
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<td>4300</td>
<td>0.014</td>
<td>0.006</td>
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<tr>
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<tr>
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<td>0.003</td>
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</tr>
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<td>4300</td>
<td>0.008</td>
<td>0.005</td>
</tr>
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</table>

of $C_1$ is almost proportional to the bottom velocity; $C_1$ varies by an order of magnitude over the range of the bottom velocities used. The corresponding variation in $C_2$ over the range of the bottom velocities used is less than 10%.

In Table I we present calculated values of $C_1$ and $C_2$. Two values of $C_1$ are given for each section: the first is for an assumed bottom velocity of 1 cm sec$^{-1}$, the second for an assumed bottom velocity of 10 cm sec$^{-1}$. The values of $C_1$ for a bottom velocity of 1 cm sec$^{-1}$ range from 0.002 to 0.014 cm$^{-2}$, for a bottom velocity of 10 cm sec$^{-1}$, from 0.017 to 0.087. The corresponding averages of $C_1$ are 0.006 for a bottom velocity of 1 cm sec$^{-1}$, 0.041 for a bottom velocity of 10 cm sec$^{-1}$. The range of $C_2$ values computed from the data is relatively narrow. In Table I, all of the $C_2$ values fall between 0.002 and 0.006 cm$^{-2}$. As was pointed out, the value of $C_2$ is not strongly dependent on the unknown bottom velocity.
3.2. Interpretation. In discussing the types of topographic control for an inertial jet flowing over the topography south of Japan, we noted that the behavior of the current appears to be different for different ranges of the value of $C_1$. A current with $C_1 = 0.005$ is in the gently steered range, where the wavelength of the topographic wave is long relative to the length-scale that is characteristic of the variations in the bottom topography; and the computed path is relatively smooth, with a very low amplitude meandering around the isobaths (Fig. 6). Recall that an increase in the value of $C_1$ to 0.04 has the effect of producing large-amplitude meandering of the current on the continental slope (Fig. 8). The figures showing the composite of all paths (Figs. 4 and 5) do not suggest that this generally occurs. Several individual paths in Fig. 4 do show large displacements across the contours, similar to the type of paths computed in Fig. 8, with the values of $C_1 = 0.03$ and 0.10; but they appear to be exceptional. The current between Kyushu and Shikoku usually remains within a relatively small range of depth. The data plotted in Fig. 15 suggest that it is unreasonable to expect that the $C_1$ values are in the range of tight topographic control, i.e., $C_1$ of the order of 1.0. This would require mean bottom velocities in excess of 50 cm sec$^{-1}$. Such high average velocities do not seem consistent with what is known about deep flow underneath western boundary currents (Schmitz et al. 1970). From the path calculations and from the composite paths in Figs. 4 and 5, there is an indication that the $C_1$ is probably in the range of gentle steering, which is consistent with a bottom velocity of the order of 1 cm sec$^{-1}$.

We re-emphasize that the data are not adequate to clearly define the nature of the topographic control of the current. Continuous path determinations are required to determine the amplitude of the meandering. Normal sections across the current cannot produce the true pattern of the current, and it is possible that there is more topographic meandering than is suggested by Figs. 4 and 5. Geostrophic calculations cannot be used to determine the type of topographic control because the velocities can only be computed relative to 800 db; there are no direct current measurements that can be used to compute the true velocity profile. This inference must be checked through direct measurements.

There is a suggestion that the value of $C_2$ differed between the periods when the meander was present and when it was absent. The $C_2$ mean value when the meander was present is 0.0040, when absent, 0.0031. This difference is statistically significant at the 95% level; it may be important in understanding the mechanism of formation of the meander. As was shown in Fig. 10, an increase in the value of $C_2$, for a fixed value of $C_1$, has the effect of moving the solutions at a faster rate down the continental slope into deeper water. A comparison of solutions with $C_2$ values of 0.002 and 0.004 shows that a relatively small change in $C_2$ produces a marked change in the position of the current over the continental slope. The possible significance of the observed difference in $C_2$ is discussed in § 4.
Unfortunately, there are no really adequate hydrographic sections across the Kuroshio west of 135°40'E. All existing profiles show rather high velocities on the inshore side of the current, so we cannot assume that a representative profile has been obtained. Although the hydrographic sections west of 135°E are not ideal for computing the value of \( C_2 \), it is useful to compare the values of \( C_2 \) computed from these sections with the values in Table I computed from adequate sections east of 135°E. West of 135°E, the meander was present in seven of these sections and absent in eight. The average values of \( C_2 \) for each of the two types of paths did not differ between the two sets of data. In both sets of data, the \( C_2 \) values were significantly higher when the meander was present.

4. Inertial Rossby-wave Solutions over the Shikoku Basin.

In §2, a variety of solutions of the path equation (4) was presented, starting with an initial position located on the continental slope south of Kyushu. These paths have physical significance as long as they remain in relatively shallow water; in that region the current has been interpreted as being gently steered by the underlying topography. Some of these paths (mathematical solutions) cross over the continental slope and extend over the
deep abyssal plain of the Shikoku Basin, where the Kuroshio flows in its meander mode. In the present section, numerical computations are made for path segments over the Basin, starting in sufficiently deep water so that assumption (iv) in §1.3 is not violated. For this investigation we introduce the hypothesis of zero-bottom velocity ($C_1 = 0$), and we consider the current as a steady inertial-Rossby wave. Recall that the inertial effect is measured by $C_0$, the $\beta$ effect by $C_2$. The paths are computed for a rather wide range of values that are suggested by the observations summarized in Fig. 5, Table I.

The results are presented in Figs. 16–18; the envelope of the observed Kuroshio meander paths is also shown. Since $C_1 = 0$, the solutions are insensitive to variations in the initial position, which is $33^\circ 45'\,N, 134^\circ 20'\,E$ in every case. Fig. 16 shows the effect of varying the planetary-wave parameter $C_2$ from 0.01 down to zero (the inertial circle) for the case of $C_0$ fixed at $-0.05$. This is a relatively large value of the curvature, as inferred from the observed paths. No attempt is made here or later in this paper to interpret the behavior of the current when it arrives at the Izu-Ogasawara Ridge. Figs. 17 and 18 exhibit the dependence of the paths on the initial curvature $C_0$ for $C_2$, fixed respectively at 0.003 and 0.004. Note the rectilinear path at constant latitude for $C_0 = 0$. Several other experiments were performed, e.g., with variations in...
Consideration of the results indicate that the flow of the Kuroshio in this region may be consistently interpreted simply as an inertial-Rossby wave. Note that this is a quantitative statement, since considerable care has been exercised in investigating the consistency of this interpretation in the light of available path and hydrographic data.


The slope paths (Fig. 4) reveal that the current almost always flows over, or in, deep water in its course from Kyushu to the Izu-Ogasawara Ridge. This occurs because of the complex and varied topography; recall particularly the short section of very narrow slope southeast of Shikoku between 134°30' E and 135° E (Fig. 1). The dynamical processes that occur when an ocean current flows from shallow to deep water are poorly understood. It is reasonable to assume that a baroclinic current will inertially cross over a short section of deep water without generating significant deep-bottom velocity. We have already observed (§ 4) that the meander paths over the Shikoku Basin (Fig. 5) can be rationalized by using a simple model with zero-bottom velocity; in any case, the bottom slopes in the Basin are so small that very large bottom velocities
would be necessary to affect the flow of the current there. We introduce, therefore, the hypothesis that whenever the Kuroshio flows into sufficiently deep water the current loses contact with the bottom. The explicit use of this hypothesis allows us to calculate long paths of the current; such long paths are compounded of segments which are gently steered by the topography in shallow water, which progress inertially over short segments of deep water, and which are inertial-Rossby waves over the deep flat Basin. The resultant computed steady-state path solutions are indicative of a mechanism that guides the current to flow in either the slope mode or meander mode.

5.1. The Turnoff Hypothesis. We assume that the detailed mechanism of the flow process in the vicinity of the steep continental slope is of only local importance. In particular, we assume that this process does not cause abrupt changes in the direction or curvature of the path. In other words, as the current loses contact with the bottom, it initially continues inertially, maintaining the curvature and direction it had received from topographic steering. For computational purposes, the process is modeled discontinuously: the path calculation is stopped at some prescribed depth, the bottom velocity is set to zero.
Figure 20. Dependence of physical path solutions upon the turnoff depth and the initial angle. The dashed path lines are for a turnoff depth \((D)\) of 3400 m and the initial angle. The solid lines are for a turnoff depth \((D)\) of 2700 m.

\((C_1 = 0)\), and a new path calculation is started from that point forward for the identical current (the same \(C_2\)), with the angle, \(\theta_0\), and the curvature \((C_0)\) for the new integration taken as given at the end point of the previously computed path segment. This procedure is referred to as “turning off” the bottom interaction at a prescribed turnoff depth, \(D\). If the computed path then shoals, the calculation is again stopped, and the bottom interaction is “turned on” for a calculation of the next path segment.

By using this model of interaction of the Kuroshio with the bottom, the physical paths for the flow from Kyushu to the Ridge can be computed for given values of the two bottom-interaction parameters, \(C_1\) and \(D\).

As a first example, we consider the physical solutions that correspond to three of the long mathematical solutions in § 2, the gently steered solutions shown in Fig. 10, sliding down the depth contours because of the \(\beta\) effect. The results are displayed in Fig. 19 for the choice of \(D = 2900\) m. Two solutions lie within the meander envelope and one within the slope envelope. Such solutions hereafter will be referred to respectively as “meander paths” and “slope paths”.

In § 2 we exhibited the dependence of the topographically steered solutions on the initial-condition parameters and on the equation-coefficient parameters.
The qualitative behavior of the physical solutions depends also upon the set of initial conditions generated at the turnoff depth. These conditions are, of course, dependent on the original set of initial conditions and on the choice of D. This dependence is illustrated in Fig. 20, which shows the effect of a variation in D from 2700 m to 3400 m and in the original initial angle from 30° to 45°. The angle variation is shown because the physical solutions are relatively sensitive to variations in this initial-point parameter. An increase in the initial angle increases the amplitude of the topographic meandering on the shelf and decreases the distance between the zeros of curvature. Thus, an increase in \( \theta_0 \) moves the current down to deep water sooner and points the current sharply across the isobaths, thus favoring a physical-meander path. The solutions in Fig. 20 demonstrate how a change in the strength (duration) of interaction of the current with the bottom, parameterized by D, can switch the current from a slope path to a meander path.

If \( C_2 \) is very small, the \( \beta \) effect is very weak and the current remains on the shelf. Simply interpreted, an increase in \( C_1 \) (\( C_2 \) fixed) tends to favor a slope solution; an increase in \( C_2 \) (\( C_1 \) fixed) favors a meander solution. However, Fig. 21 illustrates the changing of a meander solution [(1) in Fig. 20] to a slope solution by lowering \( C_1 \) (decreasing the bottom interaction). This is because the consequent decrease in amplitude of the topographically steered slope
meander yields initial conditions for the flow over deep water that points the path into the slope envelope with little curvature.

5.2. The Shadow Zone. Numerically we have obtained long steady-state solutions that simulate both the slope mode and meander mode of behavior of the Kuroshio, and we have noted that a change in the initial conditions, in current-profile parameters, or in bottom-interaction parameters can change the path from one mode to the other. Even though the mathematical model adopted is a simple one, the results of the path calculations performed for the real terrain south of Japan depend in a complicated way on the choice of the set \( S \) of the path parameters, i.e., \( S: X_0, Y_0, \theta_0, C_0, G_1, G_2, \) and \( D \). This is true even in the range of parameters that available observations suggest are pertinent to the Kuroshio.

That the Kuroshio flows steadily for several years in one mode or the other is a striking observation. We seek an explanation for this behavior in relatively simple terms. The steady-state explanation that has emerged from our numerical-path experiment is simply that, for any given model current, there is a vee-shaped region south of Honshu into which a steady path will not penetrate (mathematically, penetration is highly improbable). We call this almost-forbidden region the shadow zone. Any numerical experiment performed by varying
one of the parameters of the set, $S$, over observationally reasonable values will produce a shadow zone in the general vicinity where steady flow of the Kuroshio is never observed. We demonstrate this phenomenon in Figs. 22 and 23.

The existence of the shadow zone has a simple physical explanation. Consider any numerical experiment in which the path calculations are started on the slope southeast of Kyushu. A calculated path either remains in the shallow water all the way past Shionomisaki as a slope path or it does not. Almost always, however, the path hits the turnoff depth ($D$) somewhere south of Shikoku. The probable initial conditions for the path segment eastward of $D$ are such that the current is pointed into deep water; the curvature at $D$ varies from weakly positive values to negative values. This is so both empirically from the Kuroshio data and from an analysis of many numerical computations. Consider a current pointed due east. If the curvature is negative, the path is curved southward further away from the slope into an inertial-Rossby wave; if the curvature is positive, the current turns northward. But the slope itself broadens east of Shikoku, with the isobaths orientated more to the east than to the northeast. Thus the positively curved paths shoal and the bottom interaction again becomes operative. In summary, paths with a negative curvature are curved further away from the slope; paths with positive curvature are curved back to the slope, where they are captured by the bottom interaction and remain
in shallow water. The extension of the argument to other angles is immediate. Thus a shadow zone forms between the steady-slope solutions and the inertial-Rossby-wave solutions.

For a given numerical experiment (set $S$), there is a cluster of new initial points at depth $D$. However, a little reflection suggests, and numerical experience confirms, that the qualitative behavior of the solutions eastward of $D$ is revealed by a consideration of the paths generated by a variation in the angle and curvature conditions at a single point. For the initial point $X_0 = 133.40^\circ$E, $Y_0 = 33.25^\circ$N, which corresponds to a turnoff (average) depth ($D$) of 2900 m, the shadow zone is displayed in Fig. 22 for a current described by $C_1 = 0.0045$, $C_2 = 0.003$. The shift of the geographical location of the shadow zone with different properties of the current is illustrated in Fig. 23, which is the same as Fig. 22 except that $C_2$ is changed from 0.003 to 0.005. In Fig. 23, only the paths that define the “edge” of the shadow zone are displayed.


The pattern of flow of the Kuroshio south of Japan is particularly intriguing because the frequency distribution of the position of the current is bimodal, with each of the modes stable for periods of at least several years. This bimodal behavior appears to be unique to the Kuroshio; a similar phenomenon has not been shown in other western-boundary currents. In this paper we have looked for a mechanism that would produce a strong tendency for the Kuroshio to be located in one of two path envelopes (Figs. 4 and 5) while tending to exclude the region between the envelopes. The Kuroshio has been modeled as a steady free inertial jet that extends to the bottom over at least a portion of its path between Kyushu and the Izu-Ogasawara Ridge. Assumptions that have been made in the derivation of the model are given in § 1.4.

This study has focused on the dynamics of the Kuroshio as it flows along the western boundary. The mechanism proposed for the meander formation is dependent on the detailed behavior of the current as it flows along the bottom topography south of Japan. The local dynamics of the Kuroshio along the boundary must be understood fully in order to be able to relate the behavior of the current off Japan to any changes in the large-scale circulation. The model discussed in this paper is complement with any climatological explanation of the phenomenon.

Six parameters must be specified in order to obtain a solution to the path equation: the initial position $(X_0, Y_0)$; the initial direction of flow ($\theta_0$); the initial curvature ($C_0$) of the current; and the first and second moments of the velocity distribution integrated across the current ($C_1$ and $C_2$). In addition, to obtain long physical paths that cross from a shallow region to a deep region, we have introduced a seventh parameter, the “turnoff” depth, $D$. At this depth the bottom velocity is set equal to zero. The solutions depend on the parameters in a complicated way, and, unfortunately, all of the parameters cannot be
reliably estimated from observations. The coefficient of the vortex-line stretching term in the path equation \( C_1 \) cannot be estimated directly because of the lack of bottom-current measurements in the Kuroshio. Estimates have been made in two ways: by simply assuming a range of plausible bottom speeds, and by using indirect evidence provided by an interpretation of the available path data. Examination of the path data favors a plausible range of \( C_1 \), in which the current is gently steered by the topography (§ 2.1). The results of the calculations with the model are critically dependent upon this choice of the range of \( C_1 \). The initial curvature \( (C_0) \) and angle \( (\theta_0) \) cannot be estimated accurately from the data because there are no continuous path determinations. Values that are not inconsistent with the path data have been chosen.

An extensive exploration of the effects of changes in the parameters on the solutions has suggested two mechanisms for the production of the two modes (meander paths and slope paths of the Kuroshio).

1. The simplest explanation of change from a continental-slope path to a meander path is that there is a change from strong bottom velocity to zero bottom velocity on the continental slope off Kyushu and Shikoku. If the bottom velocity is strong \( (C_1 \) large), then the current remains on the continental slope in relatively shallow water past Shionomisaki (Figs. 4 and 7). On the other hand, if the bottom velocity on the slope is set equal to zero \( (C_1 = 0) \), then the solutions become uncoupled from the topography, and the path may be interpreted as a stationary Rossby wave. Rossby-wave solutions started on the slope southeast of Kyushu, with values of \( C_2 \) that lie in the range of values computed from the Kuroshio data, fall within the envelope of the meander paths (Figs. 5 and 11). This simple interpretation of the path computations is consistent with the data, since there are no bottom-current measurements underneath the Kuroshio. However, it seems to us that in relatively shallow water \((1000 \text{ to } 1500 \text{ m})\) it is more likely that a finite bottom velocity always exists.

2. If the current always does have a bottom velocity after it comes around the southern end of Kyushu and begins to flow northeastward along the shallow continental slope, then there is a plausible mechanism that produces a distribution of paths similar to those observed in the Kuroshio. This mechanism is based on the assumption that the current does not maintain a finite bottom velocity over the total length of its path eastward to the Izu-Ogasawara Ridge; i.e., the current will not maintain contact with the bottom as it suddenly flows from shallow water into much deeper water. If the current is steered by the bottom topography into relatively deep water and if the bottom velocity drops effectively to zero, the current will not be steered by the bottom topography unless it encounters relatively shallow water again. For the deep water over the Shikoku Basin, it has been demonstrated (§ 4) that the flow of the current is in excellent agreement with the inertial-Rossby-wave interpretation.

Examination of the bottom topography south of Japan and of the estimated
paths of the Kuroshio (Figs. 4 and 5) indicates that there is one region where the Kuroshio probably often does enter deeper water. Between 134°30' and 135°E, the slope narrows and changes orientation. Solutions of the path equation show a tendency for the current axis to be located in water that is deeper than 3000 m in the vicinity of this narrow portion of the slope. Two "turnoff" depths were used in the solutions: 2900 and 3400 m. Solutions that cross this depth from deeper water are "turned on" again, and the solutions are then steered by the topography.

It has been shown in § 5.2 that this model of the flow produces a tendency for solutions east of 135°E to be found either on the continental slope (slope path) or in the deep water over the Shikoku Basin (meander path). Over the range of parameters that seems to be most appropriate to the Kuroshio, the solutions generally fall either in the envelope of the observed slope or meander paths. Thus, the behavior of the solutions is not unlike the behavior of the Kuroshio. There is a shadow zone where the mathematical solutions tend not to penetrate; this zone is located approximately where the intermediate zone between the slope paths and meander paths were found during 1956 and 1964. The determining factors that dictate where the solutions are found are the curvature and angle of the current at the time that the bottom velocity is set equal to zero. If the solution has a negative curvature at the turnoff depth and has a small angle, the current will leave the slope and become an inertial-Rossby wave over the Shikoku Basin. On the other hand, if the solution at D has a positive curvature, then the current turns back toward the slope and recrosses the turnoff depth, where its bottom velocity \( C_t \) becomes finite; then the solution again becomes steered by the bottom topography. This type of solution remains on the slope past Shionomisaki and tends to lie in the envelope of the slope path.

Note that the shadow zone is not a region where there are no steady-state path solutions. Some combinations of the path parameters produce solutions that lie in the zone intermediate between the envelope of slope paths and the envelope of meander paths. For any given values of the parameters in the model, if one of the parameters is varied systematically while the others are held constant, the shadow zone is distinguished by a low density of solutions rather than by a total absence of solutions. There is some observational evidence that the Kuroshio is not strictly bimodal. Taft (in press) has mentioned that, in March 1963, the Kuroshio path was intermediate in position between the envelope of slope paths and the envelope of meander paths. However, at that time the Kuroshio appeared to be moving northward to a position on the continental slope after almost 3.5 years in the meander position; the present steady-state theory is inadequate to discuss this situation.

The theory of the Kuroshio path that is proposed in this paper cannot be adequately tested by presently available data. Measurements of the bottom velocity and a determination of the path of the Kuroshio, via continuous
tracking, are needed to provide a physical basis for successfully modeling the Kuroshio. Simultaneous estimates of the position of the current axis and of the velocity distribution across the current are required. Direct measurements of the bottom velocity, in both shallow and deep water, are particularly important. Such measurements would permit: establishment of the proper range of parameters in the path equation, a test of the hypothesis that the Kuroshio is gently steered by the bottom topography, and an evaluation of the turnoff hypothesis. If the model were found to be inappropriate, the availability of such data would provide a basis for more cogent modeling.

The only parameter in the path equation (4) that could be determined from existing measurements was the coefficient $C_2$. Measurements of $C_2$ when the meander was present and when it was absent differed significantly. The $C_2$ value for the Kuroshio was higher when the meander was present. In general, an increase in $C_2$, with all other parameters held constant, has the effect of moving the computed paths down the continental slope into deep water more rapidly. The effect of higher values of $C_2$ would seem to favor the formation of the meander, but the relationship between the computed paths and the parameters in the model is complicated by the nature of the underlying topography (Fig. 20).

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REFERENCES

BOLIN, BERT

FUKUOKA, JIRO

FUGLISTER, F. C.

GODFREY, J. S., and A. R. ROBINSON

HANSEN, D. V.
HAURWITZ, BERNHARD, and H. A. PANOFSKY

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