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ABSTRACT

A concept presented in this paper can account for a wide variety of observed features in the Kuroshio-Oyashio front, just east of Japan. The concept, a synthetic description, consists of two von Kármán vortex streets arranged side by side; it is referred to as a compound vortex street. Von Kármán’s kinematic theory is generalized to include all possible vortex-street arrays. Observations of sea-surface currents and mass transport in the Kuroshio-Oyashio front show reasonably good quantitative agreement with the theory; also, there is qualitative agreement between the model and the observed distributions of temperature and density in the region.

Introduction. The front where the Kuroshio and the Oyashio currents meet, just east of Japan, is one of the most interesting features of the Pacific Ocean. To the biologist it is a zone of high productivity, the site of several major fisheries, and a boundary where many diverse organisms congregate. To the descriptive oceanographer the front is a boundary between the warm saline North Pacific Central Water mass and the cold dilute Subarctic Water
mass. Dynamically, the front can be considered as a band of concentrated kinetic energy separating two regions having different potential energy. Biologically, chemically, and physically, the region displays marked contrasts and sharp gradients. Thus it is not only interesting but very complex.

This complexity makes it difficult to provide an unambiguous description of the Kuroshio-Oyashio region, even though relatively large amounts of data are available, at least for the western and southern portions of the region. Its analogue in the Atlantic, the eastern extension of the Gulf Stream off the North American coast, presents similar problems. Working with a single set of near-synoptic observations in that area, Fuglister (1955) was able to develop three dissimilar yet plausible interpretations of the temperature data. Stommel (1958: 60) noted that "the chief features are so poorly known that a serious theoretical analysis of the Gulf Stream cannot be made at present." Better descriptive models of the Gulf Stream and the Kuroshio would help to reveal the order underlying their complexity and would make it possible to conceive new theoretical approaches and improve the design of data collection at sea.

One such model, an elaboration of the vortex street concept first developed by von Kármán (1911) in his classical theoretical discussion of periodic wakes, is presented here. The model originated as an empirical concept to guide the contouring of depth, salinity, and dissolved-oxygen values on a series of charts of isopycnal surfaces in the "Oceanographic Atlas of the Pacific Ocean" (Barkley 1968).

In the following account, a variety of charts is presented as empirical evidence for the presence of a vortex-street system east of Japan. Following is an analysis that shows that the vortex street concept has some degree of quantitative theoretical validity. A discussion of the implications derived from the model in terms of theory and observation makes up the final section.

The model, an oversimplified but apparently accurate synthetic description, fits a variety of observed data to such a degree that it should probably be considered as a useful hypothesis to be tested by field work and theoretical studies.

The Compound Vortex Street. Authors considering theoretical models of meanders in ocean-current systems have largely concerned themselves with systems in which the current is represented by a single high-velocity core (Stommel 1958, Fofonoff 1962, and Ichiye [unpublished ms]). Their solutions have usually taken the form of wavelike disturbances in the axis of this core. Although such theoretical models appear to account for certain observed conditions in the Gulf Stream and the Kuroshio where these currents flow along western coastal boundaries, they do not adequately describe conditions that exist after the currents leave the coast. In these offshore areas there is considerable evidence of multiple high-velocity cores separated by weaker countercurrents (Fuglister 1951, 1955); for such areas, more complex models must be considered.
The simplest model that appears capable of representing the meanders, eddies, and countercurrents near the Kuroshio-Oyashio front is a configuration that resembles a pair of von Kármán vortex streets arranged side by side; this configuration is here termed a compound vortex street. Von Kármán vortex streets (or wakes) are a type of flow that is characterized (Goldstein 1938) by a double row of vortices, those of one row having positive (cyclonic) circulation, and those of the other, negative (anticyclonic) circulation. Fig. 1, upper panel, shows this arrangement, which has a wavelength, or pitch, \( a \), and a spacing between rows, \( h \). The lower panel in Fig. 1 (from Tietjens 1934) shows the streamlines for a single vortex street.

If we add together two identical single vortex streets, displaced \( a/2 \) unit right or left with respect to each other, so that the array is symmetrical with axes spaced \( 2h \) units apart, we obtain the streamlines of the compound vortex street shown in Fig. 2. An infinite number of combinations can obviously be obtained by varying the distance between adjacent vortex streets and their relative phase; however, the Kuroshio-Oyashio front apparently tends to assume only two configurations: a symmetrical compound vortex street similar to that in Fig. 2, or, more seldom, a single high-intensity vortex street similar to that in Fig. 1.
Figure 2. Compound vortex street.

Upper panel: Schematic diagram showing coordinate system. \( a \), wavelength; \( h \), distance between vortex rows; A, B, C, midpoints between vortex centers (see text).

Lower panel: Streamlines for a compound vortex street, from (6). Contour interval: 0.020 unit of \( \psi/k \).

Observations on Compound Vortex Streets Near Japan. To apply the vortex-street model to oceanographic data, we assume that the flow is steady, frictionless, and geostrophic, so that the streamlines in Fig. 2 might represent isotherms as well as currents, the topography of an isopycnal surface or idealized thermocline, or the dynamic topography of the sea surface.
On this assumption, in the Northern Hemisphere we can consider the streamlines in Fig. 2 as:

(i) An idealized set of isotherms, with the uppermost row of vortices representing a series of temperature minima, the next lower row representing temperature maxima, the third row representing minima, and the fourth row representing maxima.

(ii) An idealized set of isobaths representing the topography of a sigma-t surface or simplified thermocline, in which the upper row of vortices represents elevations or domes and the next represents depressions; the lower half of the figure is a mirror image of the upper.

(iii) The idealized dynamic topography of, say, the sea surface relative to 1000 decibars; here the upper row of vortices would appear as minima in the dynamic topography, the second row as maxima, etc.

We can now consider charts of the Kuroshio-Oyashio front showing temperature, topography of isopycnal surfaces, dynamic topography, and surface currents, in terms of the simple model represented by Fig. 2.

Temperature at 200 m and Surface Currents. Fig. 3 (upper panel) shows part of a chart of the temperature at 200 m for July through September 1961. The chart was prepared by the Japanese Hydrographic Division, Maritime Safety Board (Vol. 2, 1960–1964). Fig. 3 (middle panel) interprets the isotherms in terms of the compound vortex-street model. This interpretation is based primarily upon the positions of temperature maxima and minima (represented by + and − signs in the middle panel of Fig. 3) and secondarily upon the shape of the temperature contours, since somewhat less subjective interpretation is required for proper positioning of a point than of a line. Interpretation of the original temperature data in terms of the compound vortex street would require a few changes in the contours on Fig. 3, particularly near 39°N, 147–148°E, where the contours are based on a small number of widely spaced observations. If Fig. 2 were used to represent the isotherms in the upper panel of Fig. 3, the central axis would have a temperature of about 8°C and the contour interval would be approximately 0.3°C.

The model implies that westerly countercurrent flow should be present along 38°N, at approximately 143°E, 147°E, and 152°E. The corresponding current chart in the same atlas, presented in somewhat simplified form in Fig. 3 (lower panel), shows strong evidence of the westerly flow predicted by the model, with maximum values of westerly flow of 1.2, 1.0, and 0.4 knot, respectively, at the three locations. Many other features of the observed current pattern also fit the model, although a number of exceptions are evident. In Fig. 3 the wavelength of the compound vortex street is about 4.5° of longitude.

The two volumes published by the Japanese Hydrographic Division, Maritime Safety Board contain quarterly charts for the period 1955–1964. Only
Figure 3. Upper panel: Temperature (°C) at 200 m off Japan, July-September 1961.
Middle panel: Interpretation of upper panel in terms of the compound vortex-street model. Temperature maxima and minima labeled + and −, respectively.
Lower panel: GEK current observations, July-September 1961. Light arrows, velocities of less than 1 knot. Heavy arrows, velocities of 1 knot or more.
Source of data: Japanese Hydrographic Division, Maritime Safety Board (Vol. 2, 1960–1964). Reduction in scale from the original charts made it necessary to simplify the lower panel by omitting numerical velocity values, by modifying the symbols used, and by omitting a few observations of weak currents (0.5 knot or less) in coastal regions where many observations were made.
23 of these charts cover enough of the area east of Japan for inclusion of at least one wavelength (a in Fig. 2, approximately 4.5° of longitude, or 400 km) of the compound vortex street. Sixteen of these 23 charts show evidence of compound vortex-street systems as determined by the presence of at least two pairs of low-high temperature “couples” (corresponding to the inner vortices of the model) between the two major temperature gradients on charts of the 200-m temperature in the region of the Kuroshio-Oyashio front. Four other charts suggest the vortex-street phenomenon, but the data are not conclusive. Finally, three charts show no evidence of vortex streets; these charts are for the first, third, and fourth quarters of 1958; the chart for the second quarter of 1958 is considered to be inconclusive. A possible connection between the compound vortex-street system and the cold eddy near 138°E off the southern coast of Honshu is suggested by the fact that—as judged from the 200-m temperature charts—this eddy was weak from the fourth quarter of 1957 through the third quarter of 1958 (entirely absent during the second quarter of 1958), a period when no compound vortex-street system was evident east of Japan.

Density Surfaces. The upper panel in Fig. 4 shows, for the area near Japan, the topography of the 26.80 sigma-t surface for the July-September quarter (Barkley 1968). Vortices occur as isolated depressions and elevations (represented by + and − signs in Fig. 4, lower panel) in the depth-contour pattern for this surface. Fig. 4 (upper panel) also shows numerical values of the maximal or minimal depths at the center of each vortex. Note that east-west rows of vortices of the same type tend to have similar maximal or minimal depths. Averages of oceanographic station data within areas 1° of latitude by 1° of longitude form the basis for the upper panel.

The contours in the upper panel of Fig. 4 were obtained by using the model as a guide, wherever the data permitted ambiguous interpretations. It is therefore possible that some of the details of the contours may represent bias in favor of the model. However, the four east-west rows of isolated eddies would appear as such in any analysis.

The lower panel of Fig. 4 presents an interpretation of the topography of the 26.80 sigma-t surface in terms of the compound vortex street. The three or more wavelengths present have similar features that recur at intervals of 4° to 5° of longitude, or about 370 km. If Fig. 2 were used to represent the isobaths in Fig. 4, the central axis would lie at a depth of approximately 300 m and the contour interval would be about 10 m.

Dynamic Topography. Very few charts that show details of the dynamic topography of the sea surface just east of Japan have been published. Perhaps the best of them are the three presented by Ichiye (1957); these are reproduced in Fig. 5. No clearcut evidence can be found on these charts in favor of or
Figure 4. Upper panel: Average topography (depths in km) of the 26.80 sigma-t surface for the July-September quarter for all years. Averages by 1° x 1° areas. Maxima and minima in the depth of the surface are shown numerically.

Lower panel: Interpretation of upper panel in terms of the compound vortex-street model. Maxima and minima in the depth are labeled + and −, respectively.

against the presence of internal eddy pairs of the type implicit in the compound vortex-street model. However, there is some indication of such an eddy pair in the lower panel of Fig. 5, and the upper panel shows some flow with a westward component near 38°–40°N. The middle panel lacks any such eddies or countercurrent flow near the Kuroshio-Oyashio front.

It is notable that the middle panel of Fig. 5 represents a period when the cold eddy south of Honshu was almost absent whereas the upper and lower panels represent conditions at times when this cold eddy was well developed. The isotherms on the original charts (not reproduced in Fig. 5) show that a single front was present during the period represented in the middle panel,
but dual systems of fronts, similar to those shown in the upper panel of Fig. 3, were present at the times represented in the upper and lower panels of Fig. 5. These observations lend some support to the idea that a weakening of the cold
eddy south of Honshu, as in 1951 and 1958, may coincide with the presence east of Japan of a single vortex street similar to that in Fig. 1 instead of the compound vortex-street system shown in Fig. 2.

**Surface Currents.** Charts of sea-surface currents in the Kuroshio-Oyashio region (e.g., Fig. 3, lower panel) provide some additional support for the compound vortex-street model. Charts of GEK measurements made in that region by the Ryofu Maru (Japan Meteorological Agency 1964: 22, for example) often show isolated areas with westward-flowing countercurrents near 37°–39°N at intervals of 4° to 5° of longitude east of Japan, as the model suggests. These regions of westward flow seem to occur where the high-velocity cores are farthest apart, as required by the compound vortex-street model. These features are also present in the quarterly surface-current charts (Japanese Hydrographic Division, Maritime Safety Board, Vols 1 and 2); Fig. 3, lower panel, is an example.

**Theoretical Analysis.** The classical theoretical analyses of single vortex streets by von Kármán (1911) and von Kármán and Rubach (1912) as well as subsequent studies summarized by Lamb (1932) and Goldstein (1938) have provided valuable insight into the nature of fluid drag, into the behavior of periodic wakes, and into other related phenomena in real fluids, despite the idealized conditions assumed in the theoretical work. This success suggests that the analysis could usefully be extended to examine the characteristics of compound vortex streets and their implications in oceanology. However, the results must be viewed with caution when applied to the Kuroshio-Oyashio front, because factors that may be important there, such as the Coriolis force, the density stratification, and the effects of lateral boundaries and bottom topography, are not included in this theory.

The classical theoretical analysis assumes that, at a given instant, vortices are present, fully formed, in some specific geometric array. It computes the effects of each vortex on all neighboring vortices to determine an instantaneous field of flow, which in turn determines the immediate subsequent motions of the center of each vortex in the array.

Each circular vortex is assumed to have tangential velocity only, given by $V = k/2\pi r$, where $k$ is the strength (positive for cyclonic rotation) and $r$ is the radial distance from the center. This assumption yields a singular point with infinite velocity at the center of each vortex; consequently, this simple irrotational model lacks physical significance near the immediate center of each vortex.

An infinite row of identical vortices, spaced at intervals of $a$ units along the $x$-axis, sets up a velocity field whose components in the $x$ and $y$ directions are given by (Lamb 1932):
The row remains stationary with respect to the coordinate system, since all the vortex centers are located at points where the effects of vortices to the right and left cancel.

Integration of the equation for $v$ in (1) with respect to $x$, holding $y$ constant, yields the stream function for a vortex row:

$$
\psi = \frac{k}{4\pi} \ln \left[ \cosh \left( \frac{2\pi y}{a} \right) - \cos \left( \frac{2\pi x}{a} \right) \right] + \psi_0. \tag{2}
$$

The resulting pattern of streamlines is shown in Fig. 6.

Equations (1) and (2) form the basis for theoretical discussion of single and compound vortex streets, which can be considered arrays of two or more rows arranged in parallel. The velocity of translation of each row in an array can readily be computed as the sum of velocities induced at the vortex centers by all other rows. Thus, for example, von Kármán (1911) has shown that each row in a single vortex street (Fig. 1) induces a velocity in the vortex centers of the other row:

$$
U = \frac{k}{2a} \tanh \left( \frac{\pi h}{a} \right), \quad V = 0. \tag{3}
$$

Figure 6. Streamlines for a vortex row, from (2). Contour interval 0.020 unit of $\psi/k$. 

Since the signs of both \( k \) and \( h \) are opposite for the two rows, \( U \) has the same sign for both rows, and the vortex street moves as a unit with the velocity given by (3). Similarly, the streamlines of any array of vortex rows can be obtained by adding together the stream functions (2) of the component rows. For the von Kármán vortex street (Fig. 1), the streamlines are given by the stream function

\[
\psi = \frac{k}{4\pi} \ln \left[ \frac{\cosh \frac{2\pi}{a} \left( y - \frac{h}{2} \right) + \cos \frac{2\pi x}{a}}{\cosh \frac{2\pi}{a} \left( y + \frac{h}{2} \right) - \cos \frac{2\pi x}{a}} \right].
\]

(4)

We can now consider the case shown in Fig. 2, where four rows of vortices rotate in alternate directions. Addition of the velocities induced in the centers of each row by all other rows (1) gives the self-induced velocities of translation of the array:

\[
V = 0
\]

\[
U_{\text{inner}} = \frac{k_0}{2a} \left( \tanh \frac{\pi h}{a} + \tanh \frac{2\pi h}{a} \right) - \frac{k_i}{2a} \left( \coth \frac{\pi h}{a} \right),
\]

\[
U_{\text{outer}} = \frac{k_i}{2a} \left( \tanh \frac{\pi h}{a} - \tanh \frac{2\pi h}{a} \right) + \frac{k_0}{2a} \left( \coth \frac{3\pi h}{a} \right),
\]

where the subscripts \( i \) and \( o \) refer to the inner and outer pairs of rows in the symmetrical array. As in (3), capital letters refer to velocities of translation of the vortex centers.

By adding together the stream functions for the component vortex rows (2), with appropriate signs for the rotation in each case, and by displacing each row the required distances in the \( x \) and \( y \) directions (Fig. 2, upper panel), we obtain the stream function

\[
\psi = \frac{k}{4\pi} \ln \left[ \frac{\cosh \frac{2\pi}{a} \left( y - \frac{3h}{2} \right) + \cos \frac{2\pi x}{a}}{\cosh \frac{2\pi}{a} \left( y + \frac{3h}{2} \right) + \cos \frac{2\pi x}{a}} \right].
\]

(6)

with streamlines as shown in Fig. 2.

The behavior of the single von Kármán vortex street can be compared with that of the compound vortex street in Fig. 7, which shows the relationships of (3) and (5) graphically for the case where \( k = k_i = k_o \), with the dimensionless ratio \( aU/k \) as the ordinate plotted against the ratio \( h/a \) as the abscissa. In effect, the compound vortex street consists of two parts, the inner pair of vortex rows and the outer pair, each of which moves with its own
velocity, $U_i$ and $U_0$, $U_i = 0$ at $h/a = 0.24$ and is negative for lower values of $h/a$. That is, if $k_i = k_0$, the inner pair of rows is stationary when $h/a = 0.24$, moves in a direction opposite to that of the outer rows when the configuration of the array is very slender, and tends to move with the outer pair of rows when the array is broad ($h/a$ large). $U_0$ on the other hand is always positive, with a minimum value at $h/a = 0.23$. For the von Kármán vortex street, the relationship is much simpler; both rows move together, and the ratio $aU/k$ increases monotonically with $h/a$ without changing sign.

Both Fig. 7 and equations (5) show that, in general, the compound vortex street will not move as a unit, although it approaches this condition at large values of $h/a$. If, however, the ratio of $k_i$ to $k_0$ (the relative strengths of the inner and outer pairs of vortex rows) fulfills the condition

$$\frac{k_i}{k_0} = \frac{\tanh \frac{\pi h}{a} + \tanh \frac{2\pi h}{a} - \coth \frac{3\pi h}{a}}{\tanh \frac{\pi h}{a} - \tanh \frac{2\pi h}{a} + \coth \frac{\pi h}{a}}$$

(7)

which is derived by setting $U_i = U_0$ in (5), the array moves as a unit with respect to its system of coordinates.

Equation (6), as written, is valid only for the case where $k = k_i = k_0$. But, for the case where $U_i = U_0$, the equation must be modified by bringing $k$ into the argument of the logarithm as an exponent. Each of the terms in brackets would thus be raised to the power $k_i$ or $k_0$, depending upon whether it represented an inner row ($y = \pm h/2$) or an outer row ($y = \pm 3h/2$).
Equation (7) is presented graphically in the upper panel of Fig. 8. Note that $k_i/k_o$ diminishes from a value of 1.0 for large $h/a$ to zero for $h/a = 0.145$, which implies that the strength of the central vortex rows diminishes with $h/a$ and vanishes when $h/a$ reaches 0.145.

The lower panel of Fig. 8 shows the behavior of the compound vortex street when it is constrained to move as a unit, with $U_t = U_o$. Fig. 8 can be compared with Fig. 7, where instead $k_i = k_o$. Note that, if $a$ and $U$ are held constant, the strength ($k_o$) of the outer pairs increases slightly as $h/a$ decreases to about 0.34, then decreases (i.e., $aU/k$ increases) for smaller values of $h/a$. The strength ($k_i$) of the inner vortex rows decreases continually as $h/a$ decreases, reaching zero ($aU/k$ goes to infinity) at $h/a = 0.145$. In contrast, the strength of the von Kármán vortex street increases continuously as $h/a$ decreases, if $a$ and $U$ are held constant.

An important question, which will not be dealt with here, is that of the stability of the compound vortex street in response to perturbations. Domm (1954) performed stability calculations for a single von Kármán vortex street and showed that the range of conditions under which stability occurs tends to increase with time. It remains to be seen whether a perturbation analysis of the compound vortex street would yield analogous results.

Many refinements in the present theoretical analysis are possible by obvious extensions of the procedure used to obtain (5) and (6) from (1) and (2). For example, to obtain asymmetrical arrays, individual vortex rows could be displaced slightly to the east or west of the positions shown in Fig. 2. The existing fund of observations from the Kuroshio-Oyashio front, however, does not permit evaluation of the relative merits of such refinements. About all
that can be expected is a semiquantitative evaluation to see whether the front bears any resemblance to a compound vortex street such as that shown in Fig. 2.

To permit comparison between theory and observation, the ratio \( aU/k \) was computed for three points in the array from (1). The points marked A and B in Fig. 2 (upper panel) lie on the axis of the array, midway between vortices of the inner row (A) and the outer row (B) \((x = 0, y = 0 \text{ and } x = a/2, y = 0, \text{ respectively})\). At these points the north-south component of velocity is zero, and the east-west component of velocity is given by

\[
u(A) = \frac{k_0}{a} \tanh \left( \frac{3\pi h}{2a} \right) - \frac{k_i}{a} \coth \left( \frac{\pi h}{2a} \right)
\]

for point A, and by

\[
u(B) = \frac{k_0}{a} \coth \left( \frac{3\pi h}{2a} \right) - \frac{k_i}{a} \tanh \left( \frac{\pi h}{2a} \right)
\]

for point B. Measurements (Table I) indicate that the ratio \( h/a \) has a value of approximately 0.42 for the Kuroshio-Oyashio region; by substituting this value into (7), we find that the ratio of strengths of the inner to the outer pair of vortex rows, \( k_i/k_0 \), must be very nearly 0.82 for the pattern to retain its geometry \((U_i = U_0)\). Since \( k_i = 0.82 \, k_0 \), for point A,

\[
\frac{au(A)}{k_0} = \tanh \left( \frac{3\pi h}{2a} \right) - 0.82 \coth \left( \frac{\pi h}{2a} \right) = -0.46,
\]

and for point B,

\[
\frac{au(B)}{k_0} = \coth \left( \frac{3\pi h}{2a} \right) - 0.82 \tanh \left( \frac{\pi h}{2a} \right) = 0.56.
\]

By applying a similar procedure to C (Fig. 2, upper panel), which lies at a midpoint between a vortex in an outer row and the neighboring vortex in an inner row \((x = 3a/4, y = h)\), we have

\[
\frac{au(C)}{k_0} = \frac{1}{2} \left( \tanh \frac{\pi h}{a} + \tanh \frac{5\pi h}{a} + 0.82 \tanh \frac{\pi h}{a} \right) = 0.88;
\]

\( \nu \), which in this case is not zero, has a value given by

\[
\frac{av(C)}{k_0} = \frac{1}{2} \left( \sech \frac{\pi h}{a} - \sech \frac{5\pi h}{a} + 0.82 \sech \frac{\pi h}{a} \right) = 0.44;
\]

then the resultant of the two components (10) and (11) is

\[
\left( \frac{(au)^2}{k_0} + \frac{(av)^2}{k_0} \right)^{\frac{1}{2}} = 0.98.
\]
Thus the speeds at the three locations should occur in the proportions

\[ A : B : C = -0.46 : 0.57 : 1. \]

**Comparison of Theory With Observations.** Table I shows the results of measurements made from the quarterly charts in the two volumes published by the Japanese Hydrographic Division, Maritime Safety Board. The values of \( h \) and \( a \) were estimated from 200-m temperature charts, and surface currents were estimated at points A, B, and C on the corresponding charts of GEK current measurements (see Fig. 2 for the locations of A, B, and C, and Fig. 3 for examples of these charts).

The values in Table I were obtained by locating, or attempting to locate, the major vortex centers that corresponded to those of the compound vortex-street model on each chart that provided sufficient coverage (usually those for the first and third quarters of each year). The value of \( a \) was then estimated by point-to-point measurement along the axis of the array to obtain the distance between successive vortices in the same row. Values for \( h \) were then estimated as distances between adjacent rows, measured perpendicular to the axis of the array. Values for velocity of surface currents were obtained by identifying midpoints between eddy centers on each temperature chart, and then looking for GEK measurements at, or very near, the same locations on the corresponding surface-current charts. Because of the distribution of observations, Table I represents conditions in the southern half of the front almost exclusively.

Once a compound vortex street was identified as such on any given chart, the procedure for taking measurements proved fairly straightforward and unambiguous. However, the process of selecting individual eddies as "significant" and then relating them to each other as part of the model array was entirely subjective. In a number of examples the compound vortex street array, if present, was seriously distorted. A typical example appears in Fig. 3; the simplified model (middle panel) shows three pairs of central vortices, but a fourth pair can be tentatively identified in the easternmost portion of the upper panel, centered near \( 37^\circ N, 154^\circ E \). This latter eddy pair has the same temperature structure as the third (easternmost) pair identified in the middle panel—slightly more than \( 9^\circ C \) in the upper eddy and somewhat less than \( 6^\circ C \) in the lower one—but the position of this fourth eddy pair is in no way consistent with the model. Most of such anomalies appeared on the charts in the easternmost portions of the Kuroshio-Oyashio front, and it is tempting to conclude that they represent stages in the downstream decay of the compound vortex street. On this premise, such anomalies were generally ignored, and none of the measurements in Table I was based upon them. It should be emphasized, however, that the values in Table I reflect choices based upon subjective judgment, and the numerical values must be regarded as only preliminary estimates.
Table I. Geometry and current velocity of the Kuroshio-Oyashio Front.

<table>
<thead>
<tr>
<th>Year</th>
<th>Qtr.</th>
<th>Interval between Wave-length (km)</th>
<th>Ratio</th>
<th>Surface current (knots)</th>
<th>Relative speeds</th>
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<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
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<td>$\bar{a}$</td>
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Ratio of means $\bar{h}/\bar{a} = 170/404 = 0.42$. Relative mean speeds $\bar{A}:\bar{B}:\bar{C} = -0.41:0.46:1.00$. Mean of ratios $\bar{h}/\bar{a} = 0.42$. Mean of relative speeds $\bar{A}:\bar{B}:\bar{C} = -0.43:0.44:1.00$. Theoretical relationship $A:B:C = -0.46:0.57:1.00$.

It is nevertheless noteworthy that the relative mean speeds at points A, B, and C in Table I are $A:B:C = -0.4:0.5:1$ as compared with the theoretical relationship $A:B:C = -0.46:0.57:1$. This agreement is better than one might expect from observations made with the GEK, selected on the basis of a rather simple model and grouped uncritically as simple averages.

Finally, equations (8), (9), and (12) can now be used to estimate $k_0$, since $a$ and the velocities at points A, B, and C are known from Table I. The three independent estimates that result are:

$$k_0 = 5.2 \times 10^9 \text{ cm}^2 \text{ sec}^{-1}, \text{ from the mean velocity at A},$$
$$= 4.8 \times 10^9 \text{ cm}^2 \text{ sec}^{-1}, \text{ from the mean velocity at B},$$
$$= 4.9 \times 10^9 \text{ cm}^2 \text{ sec}^{-1}, \text{ from the mean velocity at C}.$$
Since the velocity at point B is based on fewer observations and is the most variable, the best estimate of \( k_0 \) is probably \( 5.0 \times 10^9 \text{cm}^2 \text{sec}^{-1} \). The value of \( k_i \) is then \( 0.82 k_0 \), or \( 4.1 \times 10^9 \text{cm}^2 \text{sec}^{-1} \).

The parameters of the sea-surface model are now completely determined. The resulting characteristics are summarized in Table II.

Table II. Sea-surface characteristics of the compound vortex-street model of the Kuroshio-Oyashio Front.

Geometry: Wavelength, \( a = 404 \text{km} \). Strength: Inner rows, \( k_i = 4.1 \times 10^9 \text{cm}^2 \text{sec}^{-1} \).

N/S spacing, \( h = 170 \text{km} \). Outer rows, \( k_0 = 5.0 \times 10^9 \text{cm}^2 \text{sec}^{-1} \).

Transport through array: \( 3.6 \times 10^9 \text{cm}^2 \text{sec}^{-1} \).

Velocity of array: \( 56 \text{cm sec}^{-1} \) (48 km day\(^{-1}\)), toward the east.

Period of array: wavelength/velocity = 8.4 days.

The properties of the compound vortex-street model listed in Table II can be divided into two categories: (i) the geometry and strength, which are based upon observations interpreted in the light of theory, and (ii) the transport, array velocity, and array period, which are deduced from the theory and which in effect constitute predictions of properties that the Kuroshio-Oyashio front must have if it is in fact a compound vortex street.

The net transport through the model in Table II was computed as follows. Transport through any part of the array can be calculated from the difference in the stream function between any two points (outside of the immediate vicinity of the vortex centers). The net flow through the array is confined to the region of near-sinusoidal streamlines between the streamlines of value \( \pm 0.4 \) and 0 (Fig. 2). Outside this region are regions of closed flow surrounding the center of each vortex; the vortices in the central rows are enclosed by ellipsoids defined by streamlines of value 0, straddling the central axis, with one vortex center near each focus; the centers of vortices in the outer rows are each enclosed by a paraboloid, with the vortex center near its focus, the paraboloid in each case having a value of \( \pm 0.4 \). Thus the net transport through the compound vortex street is 0.8 \( k \) units or \( 3.6 \times 10^9 \text{cm}^2 \text{sec}^{-1} \) if \( k \) is taken to be \( 4.5 \times 10^9 \text{cm}^2 \text{sec}^{-1} \) (average of \( k_0 \) and \( k_i \), Table II). In more common units, the total flow through the model amounts to \( 3.6 \times 10^5 \text{m}^2 \text{sec}^{-1} \) (or \( \text{m}^3 \text{sec}^{-1} \) per meter depth). In Fig. 2, the flow between adjacent streamlines is \( 0.02 k = 0.09 \times 10^9 \text{cm}^2 \text{sec}^{-1} \) (0.9 \( \times 10^4 \text{m}^2 \text{sec}^{-1} \)).

A critical comparison of the two-dimensional model with its three-dimensional prototype would require extensive information on vertical changes in velocity within the Kuroshio-Oyashio front. Although this information is not now available, enough is known to provide an "order-of-magnitude" comparison.

Geostrophic transport in the Kuroshio and Oyashio combined is estimated to be approximately \( 50 \times 10^6 \text{m}^3 \text{sec}^{-1} \) (Sverdrup et al. 1942, Uda 1964). A
compound vortex street of strength \( k = 4.5 \times 10^9 \text{ cm}^2 \text{ sec}^{-1} \) (Table II) would transport the proper volume of water if it extended uniformly to a depth of 140 m. It would also transport the proper volume if the strength decreased exponentially so as to drop by half with each 75 to 80 m increase in depth, reaching values that are one-tenth those at the surface at depths of 250 to 265 m. If the use of GEK data results in an overestimate of \( k \), or if geostrophic computations underestimate the total transport, the depth values calculated above would increase proportionately.

A velocity section across the Kuroshio and Oyashio presented by Koshlyakov (1961: fig. 2) shows that the geostrophic velocity in the vicinity of the Kuroshio-Oyashio front (35°N to 41°N, approximately) does in fact diminish rapidly with depth, from values of 25 cm sec\(^{-1}\) or more at the sea surface to one-tenth of that amount at depths of 300 to 900 m.

It is reasonable to conclude, therefore, that transport in the two-dimensional compound vortex-street model is of the proper order of magnitude. When more is learned about the transport in three dimensions in the Kuroshio-Oyashio front, it should be possible to extend the two-dimensional model to three dimensions by allowing \( k \) to diminish with depth in a realistic manner, as determined by observed decreases in current velocity with depth.

The significance of the speed and period of the theoretical compound vortex street (56 cm sec\(^{-1}\) and 8.4 days, respectively) remains to be considered. Observations suggest that the compound vortex-street array during most of the 1955-1964 decade was nearly stationary east of Japan, oscillating irregularly about an equilibrium configuration. Over longer periods of time it at least occurred with great frequency in one geographic location. Otherwise the compound vortex street would not appear on a whole series of quarterly charts (e.g., Fig. 3) and on a long-term average chart based on data taken between 1932 and 1960 (Fig. 5). These charts would appear very different if the vortex street progressed eastward by a distance of one wavelength every eight days.

We are thus left with some attractive conclusions and a set of dilemmas. The relatively simple concept of a compound vortex street accounts, in a manner that might be considered semiquantitative, for many of the major observed features of the complex Kuroshio-Oyashio front (including the distributions of temperature and density, the magnitude and direction of surface currents, and, to a degree, the volume transport). On the other hand, the kinematic theory implies that the compound vortex street should travel eastward whereas a variety of observations indicate that it tends to remain in one location. Worse still, the theoretical eastward velocity is proportional to \( k \), the strength of the vortices, and it seems certain that \( k \) in the three-dimensional prototype must diminish with depth. We are led to the astonishing conclusion that the eastward velocity of the array must also diminish with depth.

A model that accounts for so much but fails in one particular is apt to be
fundamentally correct but incomplete. The model deals with the kinematics of flow in the Kuroshio-Oyashio front and ignores the forces that govern that flow.

It seems pertinent to suggest that the eastward velocity of the model could produce a stationary array if the array's coordinate system were given an equivalent westward velocity by forces not considered in the simple theory of the model. Velocities of the proper magnitude and direction are produced by at least one class of phenomena: inertial planetary waves associated with changes in the Coriolis parameter with latitude (Moore 1963, for example) and a related class of planetary waves associated with changes in bottom topography on a rotating planet (Porter and Rattray 1964). This amounts to a suggestion that a search be made for solutions to the dynamic equations of motion having a form similar to that of equation (6).

Summary and Conclusions. At least two-thirds of the time during the 1955–1964 decade, the Kuroshio-Oyashio front east of Japan bore a marked resemblance to a symmetrical compound vortex street (Fig. 2), a type of flow consisting of two von Kármán vortex streets arranged side by side. The resemblance can be observed in quarterly charts of temperature at various depths, long-term average charts of the topography of isopycnal surfaces, quarterly charts of sea surface (GEK) currents, and, possibly, in charts of the dynamic topography of the sea surface. The wavelength was found to be about 400 km, the north-south interval between vortex rows, 170 km.

To permit quantitative comparisons, the kinematic theory first developed by von Kármán (1911) has been extended and generalized to include vortex streets of two or more rows of vortices. The theoretical model agrees well with the surface-current patterns in the Kuroshio-Oyashio front; currents at selected locations in both model and prototype flow in the same directions with the same relative speeds. Finally, if the two-dimensional model were extended to three dimensions by considering changes in current velocity with depth, volume transport of the proper order of magnitude could be obtained.

This marked resemblance between the unbounded nongeostrophic model and its prototype fails in one particular: the model predicts a substantial eastward translation of the vortex array whereas observations suggest that it remains more or less stationary. It is suggested that this contradiction might be resolved by a more complete analysis that would include the effects of Coriolis force and bottom topography.

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Barkley: The Kuroshio-Oyashio Front

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