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This paper is concerned with the dynamical processes involved in the growth of the lower frequency spectral components of wind-generated waves. Some calculations of Miles (1960) are extended, leading to the concept of a transition frequency for a given fetch as the frequency above which energy is supplied to the waves by the instability (or sheltering) mechanism and below which it is supplied by the resonance mechanism. It is shown that if the fetch and duration of the wind is not large, the transition frequency is associated with a rapid rise in the spectral magnitude, suggesting its identification with the frequency of the steep forward face of the spectrum. Reasonable confirmation of this is provided by a number of sets of published observations. It is also predicted that components whose frequencies are near the spectral maximum are oriented more strongly in the wind direction than those at lower or higher frequencies. When the fetch and duration of the wind are large, the transition frequency may be higher than the frequency of the spectral maximum. The components whose frequencies are less than the transition frequency acquire their energy by resonance with the convected surface pressure fluctuations. The resulting inferences concerning the directional distribution are discussed in relation to the two-dimensional spectrum measured during the Stereo-Wave Observation Project (1957).

1 This work was supported by the Office of Naval Research under Contract 248(56).
1. Introduction. The shape of the spectrum of wind-generated gravity waves is determined by the interplay of a number of different physical mechanisms: the resonance between water surface and convected surface pressure fluctuations associated with atmospheric turbulence; the wave growth resulting from the 'sheltering' effect of the waves; the nonlinear interactions among the wave components and the onset of dynamical instability of the water surface, leading to the formation of white-caps. In an earlier paper (Phillips, 1958a) it was shown that if the wind continues to blow, the high frequency components attain a state of saturation or statistical equilibrium, in which the net rate of energy supply to an interval of the spectrum just balances the rate of energy loss from the waves by the breaking process. In this equilibrium range, a dimensional analysis shows that the frequency spectrum is given by

\[ \Phi(\omega) = \beta g^2 \omega^{-5}, \]

where \( \omega \) is the radian frequency, \( g \) the gravitational acceleration, and \( \beta \) a dimensionless, absolute constant. There is now a good deal of observational evidence to support the relation (1.1). The earliest reliable measurements were made by Burling (1955) in Staines Reservoir, Middlesex, and his results led to a value of \( \beta \) of 7.4 \times 10^{-3}. The results of Project S.W.O.P. (1957) showed that the high frequency components of ocean spectra, taken at a very large fetch and duration, follow exactly the same relation (Phillips, 1958b). More recent observations by Kinsman (1960) and Hicks (1960) are also in reasonably good accord with the form (1.1) at high frequencies.

In this paper we turn our attention to the components of the spectrum whose frequencies lie below the equilibrium range. This part of the spectrum frequently contributes most to the mean square surface displacement, very little being contributed to the mean square surface slope; thus a realistic first approximation to the dynamical behaviour of this range of the spectrum should be provided by the linearised theories of wave generation (Phillips, 1957; Miles, 1957). According to the resonance theory, energy is supplied to the waves by the convection of turbulent atmospheric pressure fluctuations across the water surface, the most rapid growth taking place when the convection velocities and length scales of the pressure pattern correspond to velocities and wavelengths of free gravity waves. Under this mechanism, the energy in any component of the wave system grows linearly with time. On the other hand, according to the instability theory, the wind blowing over the corrugated water surface sets up a surface pressure distribution that is in phase with the surface slope and is proportional to it, so that the energy in each component grows in the exponential manner characteristic of linearised instability models.

\(^2\) The term is used here to describe the pressure variations induced by the air motion over a wavy surface (which have been estimated by Miles, 1957), without necessarily implying the presence of any region of flow separation in the lee of the waves.
It is shown in the following sections that the wave system is initiated by the resonance mechanism and that the energy in each component grows linearly until the component reaches a size whereon the instability mechanism can act efficiently, after which the energy grows approximately exponentially until nonlinear effects become important. This appears to be the sequence of events for wave components whose phase velocity is much less than the wind speed at the height of about a wavelength. However, for the components whose speed is about the same as that of the wind, the instability mechanism is very inefficient (Miles, 1960); in other words, the ‘sheltering’ effect disappears. The bulk of the energy observed in these components must therefore be provided by the resonance mechanism.

The interplay between these two mechanisms provides the theme of this paper, and the aim is to consummate the marriage of the two separate theories proposed by Miles (1960). This union enables us to explain quantitatively a number of observed properties of wave spectra and to make further predictions that lend themselves to experimental investigation.

2. The Transition Frequencies. In a recent paper, Miles (1960) has extended the analysis of wave generation by a surface distribution of fluctuating pressures set up by a turbulent wind (Phillips, 1957) to include to a first approximation the back reaction of the surface configuration upon the pressure distribution. Miles assumes that this induced pressure distribution is simply additive to the pressure fluctuations from the atmospheric turbulence and is given by

\[ p_s(x, t) = -\zeta \rho_w c(k) \frac{\partial \xi(x, t)}{\partial t}, \]  

where \( \rho_w \) is the water density, \( c(k) = (g/k)^{1/2} \) is the phase velocity of free surface waves of wave-number \( k \), \( \xi \) is the surface displacement, and the numerical parameter \( \zeta \) represents the fractional increase in wave energy per cycle resulting from this pressure distribution. This last quantity, \( \zeta \), is given by Miles’ stability analysis. When the velocity profile is logarithmic,

\[ U(y) = U_1 \log \left( \frac{y}{z_o} \right), \]

where \( U_1 = 2.5 u_* \) (i.e., 2.5 times the friction velocity) and where \( z_o \) is the effective roughness parameter, then the function \( \zeta \) has the form

\[ \zeta = \frac{\rho_a}{\rho_w} \left\{ \frac{U_1 \cos \alpha}{c(k)} \right\}^2 G \left( \Omega, \frac{U_1 \cos \alpha}{c(k)} \right). \]

In this last expression, \( \rho_a \) represents the air density, \( \alpha \) is the angle between the direction of waves propagation and that of the wind, and \( G \) is a numerically
determined function of $U_1 \cos \alpha/c(k)$ and of the additional parameter

$$\Omega = \frac{g z_0}{U_1^2 \cos^2 \alpha}. \quad (2.4)$$

The functional dependence of $G$ upon $\Omega$ is slight, so Miles suggests that little error is introduced if the value $10^{-2}$ is chosen, this being the order of magnitude expected for this parameter when the short steep waves have attained their equilibrium value.

Suppose that an infinite sheet of deep water is initially at rest and that at time $t = 0$ a turbulent wind begins to blow. If $\Pi(k, \tau)$ is the spectrum of the turbulent atmospheric surface pressure fluctuations with time delay $\tau$,

$$\Pi(k, \tau) = (2\pi)^{-1} \int [p(x, t)p(x + r, t + \tau)] e^{-ik \cdot r} dr, \quad (2.5)$$

the wave-number spectrum $\Phi(k, t)$ of the surface displacement, generated by the combined influence of the turbulent fluctuations and the induced pressure variations (2.1), is shown by Miles (1960) to be of the form

$$\Phi(k, t) = \frac{f(t, m)\int_0^\infty \Pi(k, \tau) \cos \omega \tau d\tau}{2 \omega^2 c^2(k)}, \quad (2.6)$$

here

$$\omega = (g k)^{1/2},$$

$$m = \frac{1}{2} \zeta k c(k),$$

$$f(t, m) = \frac{e^{2mt - 1}}{2m}$$

$$= t \{ 1 + mt + O[(mt)^2] \}. \quad (2.7)$$

The expressions (2.6) and (2.7) are of considerable interest. If $mt \ll 1$, $f(t, m) \approx t$, and (2.6) reduces to the expression given earlier (Phillips, 1957) for the generation of waves through the resonance mechanism by the atmospheric turbulent pressure fluctuations alone. The components of the spectrum grow linearly with time. For a given wave-number $k$ (in other words, for a given $m$), as the wind duration increases the rate of growth increases from this linear behaviour to an exponential one characteristic of an instability, the transition occurring at a wind duration $t$ such that $mt$ is of order unity. Therefore we can define a transition wind duration for each wave-number such that

$$T_t(k) = m^{-1} = \left\{ \frac{1}{2} \zeta k c(k) \right\}^{-1}, \quad (2.8)$$

about which the mechanism of wave generation changes from resonance to instability and the time history from linear to exponential. An interesting and
important property is that $T_r$ is independent of the magnitude of the atmospheric turbulent pressure fluctuations on the water surface.

Unfortunately, it is very difficult to make reliable observations under the duration-limited conditions envisaged by this theory. A number of measurements have been made, however, in fetch-limited situations, where an approximately steady wind blows from a clearly defined shore line for a time sufficient to ensure that the wave field is statistically steady in time. This fetch-limited situation is complementary to the duration-limited case discussed in the theory, and there exists a simple transformation between the two. For a given Fourier component one might expect (a proof is given by Phillips, 1958d) that a duration $T$ is dynamically equivalent to a fetch $F$ given by

$$F = \frac{1}{2} c(k) T$$

for linearised waves; here $\frac{1}{2} c(k)$ is the speed of energy propagation, or the group velocity. The expression (2.9) is valid when $F \gg \lambda$, the wavelength of the component in question, and this condition is always amply satisfied in our considerations. Observations at a fixed fetch are conceptually equivalent to observations over an ensemble of duration-limited wave fields originating at some previous instant. For a fixed fetch, the equivalent duration is different for different wave-numbers, $k$, or frequencies, $n$, since the group velocity is a function of $k$ or $n$.

It is convenient, then, to translate the expression (2.8) into one giving the fetch $F_t$ at which transition occurs for a given wave-number. From (2.9), we have

$$F_t(k) = \frac{\frac{1}{2} c(k)}{m} = \left\{ \zeta k \right\}^{-1},$$

so that the number of wave-lengths travelled by a component of the wave system before transition is simply

$$\frac{F_t(k)}{\lambda_t} = (2\pi \zeta)^{-1},$$

which, from (2.3) is a function of $U_t \cos \alpha/c(k)$. The continuous curves of Fig. 1 were derived from Miles’ (1960) calculations; $\Omega$ is taken as $10^{-2}$, $\Omega_0/\Omega_0 = 1.2 \times 10^{-3}$, and the ‘wind speed’ $U$ is defined as $9 U_t$ to correspond to the wind speed as usually measured several meters above the surface. It is clear that for a given frequency and an increasing fetch, the transition occurs earliest when $\alpha = 0$, or for the wave components travelling in the direction of the wind.

These relations can be interpreted inversely. For a given fetch, there is a transition frequency below which energy is supplied to the waves by the
atmospheric turbulent pressure fluctuations and above which (for components travelling in the wind direction) energy is supplied by the 'sheltering' effect associated with the instability.

Let us now inquire whether this transition might be related to some observable property of the spectrum of wind-generated waves. The nature of the transition is such that the wave growth begins to accelerate rapidly, until nonlinear or surface stability effects become significant. Now, a very characteristic property of frequency spectra measured at a point is that the

Figure 1. The number of wavelengths of wind-generated waves before transition.
spectral density rises quite abruptly from low frequencies to a maximum and then falls off more gradually (approximately as $\omega^{-5}$) as the frequency increases. As the fetch increases, the frequency at which this forward face is found decreases; or conversely, if we consider one particular frequency of the spectrum, for increasing fetch, the value of the spectrum rises rapidly with increasing fetch as the steep forward face moves past our observation frequency to lower values. This is exactly the property expected as a result of the transition from growth dominated by the resonance mechanism to one dominated by an instability; and it strongly suggests an identification between the frequencies of the steep forward face of the spectrum at various fetches and the transition frequencies that emerge from the theory.

It should perhaps be pointed out that the theory contains no adjustable parameters, so that any discrepancies between theory and experiment must be attributed to a lack of correspondence between the conditions of observation and those envisaged by the theory.

3. Some Observations at Short and Moderate Fetches. Let us therefore compare the transition frequencies inferred above with the frequencies of the steep forward face measured in recent observations. Three sets of measurements, made at short or moderate fetches, appear to be reasonably suited to our purpose; the minimum requirements are moderately well defined conditions of observation and the measurement of detailed spectra. These are the results of Burling (1955), Kinsman (1960) and W. J. Pierson. A disadvantage of all three is the lack of detailed wind measurements, though Kinsman (1960) does give a little information on the profiles.

The frequencies at which the spectral density rose abruptly were taken from the individual spectra, of which Fig. 2 is a representative sample. Since the rise was abrupt in almost all cases, there was little possibility of significant subjective error. From these frequencies, the corresponding phase speeds and wavelengths were calculated. Since transition occurs first for the components travelling in the direction of the wind ($\alpha = 0$), the fetch was in each case taken as the distance to the upwind shore.

The data taken from these three sources are shown in Fig. 1, where the observed frequencies of the steep forward face are certainly of the same order of magnitude as the predicted transition frequencies. The scatter however is considerable and the range of values of $F_t/\lambda_t$ over which observations were made is limited. There are probably a number of reasons for the large scatter of the observational points, despite the considerable precautions taken in each case to ensure that the wind was approximately steady in magnitude and direction. In the first place, Miles' (1957, 1960) calculations for the function assume that the wind velocity profile is logarithmic, whereas Kinsman's (1960)

3 Mr. Pierson's measurements have not yet been published. We are grateful to him for his permission to use these data.
Figure 2. A typical spectrum of wind-generated waves at short fetches: Kinsman's (1960) record 093. The broken line indicates the frequency of the steep forward face.

Wind data, taken simultaneously with the wave data, show that in many cases the actual profile differed considerably from the logarithmic form. The analysis of Benjamin (1959) shows that different (but not implausible) velocity profiles can result in a variation of order 50% of the theoretical curve for $F_c/\lambda$, at a given $c/U$. Furthermore, the analysis envisages the motion as starting from rest. If, in the experiments, there is any background wave motion near the windward shore, the transition for a given frequency will occur at a shorter fetch. Since the wave growth under the influence of the turbulent pressure fluctuations is relatively slow under these conditions (small $c/U$), even a small background would have a significant effect and would result in the observational points lying below the theoretical curve, as indeed do most of Burling’s and Kinsman’s results.

Pierson's measurements were made in a water tank mounted in a large wind tunnel. Detailed velocity profiles near the water surface were not measured. At these short fetches and low values of $c/U$, the function $\zeta$ depends critically upon the form of the profile very near the water surface. Small

4 If $u(y)$ is the wind profile, $\zeta$ is approximately proportional to $u''(y)$ where $y$ is the height where $u(y) = c$ (Miles, 1957).
variations in the profile will therefore result in very different transition frequencies, since the observations depend very sensitively on a flow property that is very difficult to control. The considerable scatter in Mr. Pierson’s observations can probably be attributed to this cause, but it is noteworthy that his points do straddle the calculated curve of Fig. 1.

It seems, then, that in view of the uncertainties inherent in the observations, the agreement is as good as can be expected; and the interpretation of the frequency of the steep forward face of the spectrum under these conditions as the transition frequency is likely to be correct. Since almost all of the wave energy is at frequencies higher than the transition frequency, it follows that most of the energy has been supplied to the waves under these circumstances, by the ‘sheltering’ or instability mechanism or by the pressure variations induced by the flow of air over the waves. These circumstances are by no means universal, as we show in the next section, but, on the basis of the theory and the observations so far available, we might expect similar behaviour for small or moderate fetches (the experiments described above range to about 1500 m) for all except very light winds. Our conclusions here seem to provide the correct explanation for Cox’s (1958a) failure to observe the directional peaks associated with the resonance theory in a tank experiment. Since \( c/U \) was small in his experiment, the resonance mechanism only provided a trigger for the growth of the instability, so that the resonance maxima at angles \( \cos^{-1} c/U \) to the wind direction would be entirely obscured. Earlier remarks on this subject (Phillips, 1958c: 228) are probably of lesser significance.

The conclusions reached here make possible a further prediction. The transition to an exponential rate of growth occurs first when \( \alpha = 0 \), irrespective of the form of the turbulent pressure spectrum. If this results in a rapid growth of these spectral components and a steep forward face of the wave spectrum, then it follows that the components having frequencies near the spectral maximum should have propagation directions oriented strongly towards \( \alpha = 0 \). For frequencies greater than the ones near the spectral peak, transition will have occurred for components whose propagation directions cover a wider angle. These components also grow rapidly and may attain their saturation value; in any event, the directional distribution of the components at these frequencies will, according to these ideas, be much broader. This prediction appears to be in qualitative accord with casual observations of wind-generated waves at short or moderate fetches. A detailed experimental study along these lines has not yet been undertaken, though it seems feasible, and it would be decisive in establishing the significance of this transition.

4. Observations at Large Fetch and Duration. It is fairly evident that the discussion of the previous section does not include all cases of interest. If the fetch or duration of the wind is very small, the instability or sheltering mechanism has no opportunity to operate at any frequency, and any waves gener-
ated must be attributed to the resonance mechanism. The early observations of Roll (1951) at fetches of a few meters are consistent with this remark.

It seems paradoxical at first that the resonance mechanism is capable of contributing significantly also when fetch and duration of wind are both very large. The reason is, that for wave components whose phase velocity is approximately equal to the wind velocity, the 'sheltering' effect is very weak; Fig. 1 shows that the transition fetch is of order $10^4 \lambda_t$ or greater. On the other hand, the resonance mechanism operates most effectively under these same conditions, so that the spectral density for these components, increasing linearly with fetch or duration, may approach the saturation value before the transition fetch or duration is attained. Components travelling at the same speed as the wind, or faster, are unable to extract any energy from the wind by the instability mechanism; indeed, they may even supply energy to the wind by a 'negative sheltering', so that, if any such components are observed, they must be attributed either to the resonance mechanism or to the effects of 'sheltering' by intermittent gusts of wind with velocity much greater than that of the mean wind.

A number of observational studies have suggested that when the fetch and duration of the wind are both large, components for which $c/U$ is of order unity may contribute significantly to the mean square wave height. The ideal situation would be one in which the wind field is approximately steady and uniform over a wide expanse of open ocean, but few of the observations available at present approach these requirements. The most suitable for our purpose seem to be the results of the Stereo-Wave Observation Project (1957). The waves were generated primarily by a wind of some 20 knots which was approximately steady for about 18 hours; the frequency spectrum, measured by a wave pole, is shown in Fig. 3. The transition frequency $\omega_t$ under these conditions is shown in this diagram and falls at a frequency greater than that of the spectral peak.

The simplest interpretation of the dynamical processes involved is that, for frequencies less than $\omega_t$, including those near the spectral peak, the wave energy has been acquired from the wind by resonance with the atmospheric turbulent pressure fluctuations; and for frequencies greater than $\omega_t$, energy is currently being supplied by the 'sheltering' mechanism or by the wave-induced surface pressure fluctuations. For frequencies greater than $\omega_t$, the spectrum is very near its equilibrium or saturation value, so that the energy being continually supplied by the latter mechanism can result in little further wave growth. Some of it is doubtless transferred to other spectral components by nonlinear interactions, but most will ultimately be lost to the waves through the formation of white-caps.

The alternative to this is to postulate that the forward face of the S.W.O.P. spectrum is still the result of the transition to an unstable rate of growth. The equivalent fetch for this spectrum is of order 200 miles so that, if $\lambda$ is the
Figure 3. The frequency spectrum for the S.W.O.P. wave-pole data. The solid curve represents the equilibrium range asymptote, the broken line the transition frequency \( w_i \).

The wavelength of the components near the forward face, \( F/\lambda \approx 4 \times 10^4 \). Fig. 1 shows that the transition frequency is therefore such that \( c/U = 0.98 \). This in turn implies that the wind speed \( U \) used in the upper scale of Fig. 3 was too small. In order that \( c = U \) for frequencies near the forward face, it is necessary that the wind speed be greater by some 70% over the entire generating area than the authors of the S.W.O.P. report believed. The existence of such a large error as this strains the credulity of even the present authors, and this alternative hypothesis seems untenable.

It appears to be inevitable, then, that our first interpretation is the correct one and that the wave energy present in the components of frequency less
than $\omega_t$ has been acquired by the resonant interactions between the waves and the convected turbulent pressure fluctuations on the surface. There is also other evidence that points to the same conclusion. If these components are generated by the resonance mechanism, then the directional distribution of the components at a fixed scalar wave-number $k$, or a fixed frequency, would be expected to possess maxima at angles $\cos^{-1}\{c(k)/U\}$ to the wind direction. For these components, the resonance condition is satisfied exactly (Phillips, 1957) and the growth is most rapid. The two-dimensional spectra as measured in the S.W.O.P. project do indeed show directional maxima close to the predicted angles (Phillips, 1958b), over frequencies of order $\omega_t$ and less. Although, as Cox (1958b) pointed out, the existence of these maxima is not by itself completely conclusive, it now appears to be quite consistent with these new results. The presence of the directional maxima is predicted by the theory in a natural manner and does not require postulation of the existence of two distinct wind systems of relatively high velocity.

The transition frequency $\omega_t$ shown in Fig. 3 has another interesting aspect. Near the transition frequency, the rate of energy transfer from the wind to the waves is greatly enhanced by the operation of the sheltering mechanism, although the magnitude of the spectrum cannot increase beyond the equilibrium range value because of the requirements of surface stability. The interesting possibility suggests itself that the small rise in the S.W.O.P. spectrum at this frequency represents the remnant of the steep forward face found at smaller fetches and durations that has largely disappeared through the growth of lower frequency components by the resonance mechanism.

The roles played by the various dynamical processes, inferred by examination of the S.W.O.P. data in the light of the wave generation theories, are expected to be characteristic of those operating when the fetch and duration of the wave field is large and when components are present that travel at about the same speed as the mean wind. It is difficult to estimate at this stage just how large the fetch and duration must be in order that the transition frequency lie above the frequency of the spectral peak. This depends on the magnitude of the turbulent surface pressure fluctuations that generate wave components whose phase speed is near the wind speed, and about these we have little information. Among other effects, we might expect that the stability of lower layers of the atmosphere would be important, since unstable conditions result in higher turbulent intensities, hence larger surface pressure fluctuations.

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