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OBSERVED AND COMPUTED PATH RADIANCE IN THE UNDERWATER LIGHT FIELD

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ABSTRACT

This paper presents experimental data on changes in underwater radiance in zenith and nadir directions as a function of depth for both overcast and clear sunny lighting conditions at the surface. The equation developed by Preisendorfer (1957) to describe path radiance along any path is then used to predict observed changes, and predicted results are compared with the observed data.

INTRODUCTION

Recently Preisendorfer (1959) published theoretical proof of the existence of characteristic diffuse light in natural waters. This proof formalizes the early conjectures of Jerlov and Liljequist (1938), Whitney (1941a, 1941b), et al., which were based on experimental data obtained by Jerlov in the Baltic and by Whitney in Wisconsin lakes and in the Atlantic Ocean near Woods Hole, Massachusetts.

The concept of characteristic diffuse light, or asymptotic radiance distribution as it has more recently been called, sometimes requires drastic changes in the distribution of radiance as a function of depth. Highlights in this distribution must be attenuated and dark areas must be filled in with space light. Jerlov (1951) has estimated that these changes are completed at about 300 m in the ocean while Tyler's (1960) data have shown that the changes are nearly complete at 60 m in the water of Lake Pend Oreille. Attempts to study these changes by measuring downward, upward, and horizontal “illumination” (Poole, 1945) have apparently not been successful, probably due to the integrating characteristics of the optical collector used. Jerlov and Liljequist (1938) and Jerlov (1951), however, have given comparative radiance data at 5 m and 15 m which show attenuation in the sun's direction as well as an increase in “vertical intensity”
with depth. Whitney's (1941b) radiance data at three depths (3 m, 6 m, 9 m) have also demonstrated attenuation in the direction of the sun and a progressive increase in "light intensity" in the vertical direction as depth increases; his data are especially interesting because they demonstrate that the rate of change of the zenith radiance ("light intensity") with depth is decreasing. More detailed measurements with monochromatic light (Tyler, 1958), made at depths of 2.5 cm, 3 m and 6 m, demonstrate the complex nature of changes in radiance that are taking place in the upper layers of the ocean.

The theoretical treatment of radiative transfer in hydrosols, worked out by Preisendorfer, has contributed greatly to a clear understanding of the physical nature of radiative transfer in a multiple scattering and absorbing hydrosol (nonisotropic). The theory can be used to predict changes in radiance along a fixed path of sight underwater.

This paper presents monochromatic data for fixed paths of sight, illuminated by constant sky conditions, over a depth range from c. 0.6 to c. 60 m. The equation for path radiance will be given from Preisendorfer (1957) and the observed radiances will be compared with those obtained by computation.

**EXPERIMENTAL**

A radiance measuring device (Tyler, *et al.*, 1959) having a solid angle of acceptance of 6.6° was oriented vertically and lowered into optically deep homogeneous water. The water involved in this experiment is described in detail by Tyler (1960). For the bandwidth under consideration the total attenuation coefficient $\alpha$ was 0.442/m, the diffuse attenuation coefficient at 29 m was $K = 0.169$/m, and the absorption coefficient was $a = 0.117$/m. Since the transfer of radiant energy within a hydrosol depends on physical factors only, not on chemical or biological considerations, the data presented here can be applied equally well to any hydrosol having physical properties which are identical with those of the water used in this research. For example, the water found by Nielsen and Jensen (1957) at their Station 109 in the South Atlantic off Angola is apparently very similar to the water described here. A Wratten No. 45 filter combined with the sensitivity of the multiplier-phototube itself limited the half-bandwidth to about 64 $m\mu$ with peak value at 480 $m\mu$. Radiance
Figure 1. Observed data for zenith (Curve A) and nadir (Curve B) path radiances as a function of depth for a clear sunny day, sun altitude 56.6°.

measurements were obtained in the zenith and nadir directions simultaneously.

Observed data for zenith and nadir directions on a clear sunny day with sun altitude of 56.6° are shown in Fig. 1. Referring to Curve A, if the thickness of the water film between the measuring device and the surface could be made zero, then the instrument would measure the radiance of the zenith sky. As the instrument is submerged the radiance in the zenith direction increases because the increase in light due to scattering along the path is greater than the decrease in light due to attenuation along the same path. At 3.7 m these two effects just balance each other, with the result that
the slope of the curve at this depth is zero. Below 3.7 m the negative slope of the curve, which at any point is equal to the instantaneous value of the attenuation coefficient for radiance (radiance k value), is increasing and is in fact approaching the asymptotic value, k00, discussed by Preisendorfer (1959).

In the nadir direction (Curve B), with the measuring device at the surface, the reading is at first influenced by the back scattering of the sun’s rays, which is indicated in Curve B by its initially high slope. This effect is quickly lost. As depth increases the negative
slope of the nadir curve decreases, again approaching $k_{00}$ as an asymptotic value.

When the slopes of the two curves in Fig. 1 are compared critically at 60 m they are found to differ slightly. This means that the region of asymptotic distribution (Preisendorfer, 1959) was not reached in this experiment.

Experimental data of this same kind, obtained under an overcast sky, are shown in Fig. 2. In this case both nadir (Curve C) and zenith (Curve D) radiance readings yield relatively high slopes at the surface, with the slopes of both curves decreasing with depth.

In Fig. 3 the zenith readings for the two lighting conditions have been adjusted to have the same radiance value at about 60 m. Thus
Fig. 3 can be used to represent a situation (in homogeneous water) in which a dark object and a light object both appear in the initial radiance distribution for the scene. The directions in which these objects occur are not specified, nor is the initial depth. From Fig. 3 the general principle of radiance attenuation can be seen at a glance; radiances with magnitudes that are relatively too large to conform with the asymptotic radiance distribution are attenuated rapidly whereas those with magnitudes that are relatively too small are attenuated slowly, or even enhanced. The net effect of this process is to inexorably return the distribution to its asymptotic shape. It follows that there must be a value of radiance which will be attenuated linearly over all depths. Its magnitude will depend on the values of \( \alpha \) and \( s \) for the hydrosol. It is approximated in Fig. 3 by the intersection of the dashed line with the ordinate. The slope of the dashed straight line is of course equal to \( k_{00} \).

**THEORETICAL PREDICTIONS**

Preisendorfer (1957) has developed a general equation to describe the path radiance along any path as a function of its length:

\[
N_r(Z, \theta, \varphi) = N_0(Z, \theta, \varphi) e^{-\alpha r} + \frac{N_* (Z, \theta, \varphi)}{\alpha + K \cos \theta} \left[ 1 - e^{-(\alpha - K \cos \theta) r} \right]. \tag{1}
\]

In this equation \( N_r(Z, \theta, \varphi) \) is the radiance of a path observed in the direction \( \theta, \varphi \), from a depth \( Z \). The first term on the right is the transmitted radiance from the source (e.g., a patch of zenith sky). The second term represents the space light which accumulates in the path by processes of scattering and attenuation.

\( N_0(Z_t, \theta, \varphi) \) is the inherent radiance of the source at depth \( Z_t \) observed in the direction \( \theta, \varphi \);

\( \alpha \) is the total attenuation coefficient of the hydrosol;

\( r \) is the length of the path;

\( N_* (Z, \theta, \varphi) \) is the path function in the direction \( \theta, \varphi \) from depth \( Z \) (i.e., the radiance per unit length along the line of sight, which is generated by the scattered light only);

\( K \) is the diffuse attenuation function;

\( \theta \) is the angle between the vertical and the direction of the path; and

\( \varphi \) is the azimuth angle.
For a path of sight directed at the zenith, the angles $\theta$ and $\varphi = 0^\circ$ and $\cos \theta = 1$, $N_0(Z_t, \theta, \varphi)$ can be interpreted as the radiance of the zenith sky ($N_0$) as seen from just below the water, and the path length, $r$, becomes equal to depth $Z$. Introducing these conditions into (1) and dropping the parenthetical notation for $\theta$ and $\varphi$, we have the expression

$$N(Z) = N_0 e^{-\alpha z} + \frac{N_*(Z)}{\alpha - K} \left(1 - e^{-(\alpha - K)z}\right),$$

which should predict Curve A of Fig. 1 and Curve D of Fig. 2.

For a path of sight directed downward, $\theta = 180^\circ$, $\cos \theta = -1$, $\varphi = 0^\circ$, $N_0 = 0$ (i.e., the bottom is so far away that only space light needs to be considered) and $r = 0$.

Substituting these conditions into (1) and simplifyfying yields

$$N(Z) = \frac{N_*(Z)}{\alpha + K}.$$ 

This equation should predict Curves B and C of Figs. 1 and 2 respectively.

During the experiment described here, all of the unknowns of (2) and (3) were measured except $N_*(Z)$. In order to compute $N(Z)$ as a function of $Z$ from (2) and (3) it is necessary to first evaluate $N_*(Z)$. The following methods were used.

**Method 1.** One procedure for evaluating $N_*(Z)$ is based on the fact that $N_*(Z)$ is generated by scattering and is thus diffuse light which will be transmitted exponentially in accordance with

$$N_*(Z) = N_*(0) e^{-KZ}.$$ 

A single value of $N_*(Z)$ can be computed from experimental data by means of (2), and from this $N_*(0)$ can be determined by using (4). The resulting value of $N_*(0)$ can now be substituted into (2) to give

$$N(Z) = N_0 e^{-\alpha z} + \frac{N_*(0) e^{-KZ}}{\alpha - K} \left(1 - e^{-(\alpha - K)z}\right),$$

which can be used to compute $N(Z)$ as a function of $Z$ over the depth range.
Method 2. Using the form of the equation of transfer for radiance given by Preisendorfer (1959), we have

$$\frac{dN(Z, \theta, \varphi)}{dr} = -\alpha N(Z, \theta, \varphi) + N_*(Z, \theta, \varphi);$$  \hspace{1cm} (6)

we again apply the conditions for vertical measurements, i.e., \( r = Z, \theta = \varphi = 0 \) to obtain

$$\frac{dN(Z)}{dr} = -\alpha N(Z) + N_*(Z).$$  \hspace{1cm} (7)

Figure 4. Comparison of observed (A) and computed (A') values of the zenith path radiance and of the observed (B) and computed (B') values of the nadir path radiance for clear sunny lighting conditions.
Figure 5. Comparison of observed (D) and computed (D') values of the zenith path radiance and of the observed (C) and computed (C') values of the nadir path radiance for overcast lighting conditions.

In the zenith data for the clear sunny day, \( [dN(Z)]/dr \) is 0 at \( Z = 3.7 \) m. Thus \( N_z \) can be computed for this depth from

\[
N_z(3.7 \text{ m}) = \alpha N(3.7 \text{ m}).
\]

(8)

Using observed values of \( \alpha, K, Z, N_0, \) and \( N(Z) \), a series of points were computed giving the predicted zenith and nadir radiance as a function of depth. These points have been plotted to give curves A', B', C', D' in Figs. 4 and 5. Fig. 4 compares the results of the com-
putation with observed values obtained on a clear sunny day; Fig. 5 compares the results of computation with observed data for an overcast day.

DISCUSSION

The differences in slope between observed and computed curves is believed to be the result of small inaccuracies in the measurement of $\alpha$ and $K$. For example, an average experimental value of $K$ was used in the computations whereas theory calls for the asymptotic value. If a value of $K$ closer to the asymptotic value had been used, then the computed zenith curves would have been much closer to the observed curves. Several points have been computed with an extrapolated value of $K$ for the asymptotic value. These points, plotted and circled in Figs. 4 and 5, indicate the result that might have been obtained if the asymptotic value of $K$ could have been measured.

The curves shown in Figs. 1, 2, 4 and 5 are on the same relative basis and can be intercompared. From Curves A and D it can be seen that, although there was generally less light in the water on the overcast day, the radiance from the zenith direction was actually greater on the overcast day down to a depth of about 10 m.

The approach to asymptotic radiance distribution can be seen in a number of ways. In both Fig. 1 and Fig. 2 the slopes of the zenith and nadir curves are approaching a constant value ($k_{00}$). Also, the ratio of zenith to nadir radiance ($\log N_{\text{zenith}} - \log N_{\text{nadir}}$) is approaching the same constant value for both the clear sunny day and for the overcast day.

The lack of dependence of the shape of the asymptotic radiance distribution on the surface lighting is seen in the facts that the slopes of the curves for the sunny day are approaching those for the overcast day and that the ratio of zenith to nadir radiance approaches the same value (approx. 30) for both days.

This comparison of observed and computed path radiance is one of a series of experiments designed to validate the nonisotropic, multiple scattering theory of Preisendorfer and to prove methods for measuring the specified water properties. In general the results of these experiments have consistently demonstrated the validity of this theory and have indicated its usefulness in handling problems relating to underwater light.
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