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MEASUREMENTS OF SLOPES OF HIGH-FREQUENCY WIND WAVES¹

BY

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ABSTRACT

Water-surface slopes due to wind waves have been measured in a laboratory wind-wave tunnel. In all measurements, nonlinear effects are important for high-frequency wind waves; nonlinear effects noted are: increase of phase velocity due to finite amplitude of gravity waves, trapping of ripples on the leading slope of gravity wavelets, and limitation of slopes after the explosive growth of waves beyond a critical fetch.

The mean square water-surface slope observed in the tunnel showed a complicated behavior as a function of wind speed. The slope increases rapidly above a sharply defined minimum wind speed for wave generation; the minimum wind speed decreases with increasing fetch. There is no large effect of air-water temperature difference.

The frequency spectrum of slopes shows (for light winds) two peaks separated by a minimum near the frequency corresponding to waves of minimum-phase velocity. As the wind increases, the frequency difference between peaks tends to increase.

Measurements at three frequencies have shown changes of phase velocities of ripples as a function of wind speed that amount to as much as 30% of the theoretical phase velocity appropriate to still water and no wind; much smaller changes are noted for gravity waves. In general, the highest velocities are associated with the strongest winds.

Comparisons with Jeffreys' (1924-25) and Phillips' (1957) theories of wave generation show disagreements at short fetches.

1. INTRODUCTION

The friction of wind blowing over the sea presents an unusually difficult subject to discuss because the roughness of the surface is not

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This article by Dr. Cox and the one by Dr. Phillips which follows, both pertinent to the same area of interest, were submitted at approximately the same time, and since it is the policy of this Journal to encourage discussion, I asked each author if he would be willing to prepare a constructive comment on the other's article. Both authors gladly agreed to my proposal, and the Comments which appear after each article resulted. To each author, many thanks.

Y. H. Olsen
only caused by but is the cause of the pattern of flow in the air above the surface and in the water beneath. There have been several attempts to deduce properties of the large roughness elements (ocean waves) from various ad hoc hypotheses concerning the nature of the interactions between wind and water, but no great success has attended these efforts. (See Ursell, 1956; 216–248, for a discussion of these attempts and their limitations.) It is with the hope of finding some of the effective forces that a study has been made of the smallest roughness elements on the sea surface: high-frequency waves and ripples.

The first problem is to describe in some way, statistical or otherwise, typical shapes of the sea surface and to find how this shape changes with time. In representing the shape of the water surface, it has been found convenient to use the slope of the water surface rather than the more usual elevation from mean water level as the parameter to be studied. This is done because high-frequency waves, although of low elevation, have large slopes. It also turns out that rather simple apparatus can be used to detect slopes of the water surface.

Suppose that the elevation of the water surface is \( z(x, y) \), where the horizontal axes \( x, y \) are directed toward the mean direction of the wind and cross wind, respectively. Then

\[
\begin{align*}
    s_x &= \frac{\partial z}{\partial x}, \\
    s_y &= \frac{\partial z}{\partial y}
\end{align*}
\]

represent the up-down wind and cross-wind components of slope, respectively. In the experiments to be described below, the principal emphasis has been laid upon measurements of the up-down wind component of slope in time and space.

2. APPARATUS

Wind waves were produced in a wind-water tunnel (Fig. 1). A centrifugal blower driven by a 3/4-hp motor draws air at speeds up to 12 m sec\(^{-1}\) through the tunnel. Observations of waves are made through any of the glass ports by means of the photocell telescopes illustrated in large scale. For measurements at a single point on the water surface, a single telescope is used; for simultaneous measurements at two points, two telescopes are used.

The optical principle upon which slopes are measured is that of refraction of light at the air-water interface. Consider a bundle of rays which passes through the pinhole and terminates on the sensitive surface of the photocell. Since the telescope is focussed on the water surface, all of the rays must have passed through, and been refracted at,
a small spot on the water surface. The region of origin of rays on the "wedge" (cross-section indicated at 8, Fig. 1) then depends primarily on the up- or down-wind component of tilt of the water surface at this spot. The wedge was so constructed that its transmission of light varied linearly from almost nil at the thick end to almost unity at the thin end. Hence brightness of light entering the photocell was proportional to the up-down wind component of slope of the water surface. By rotating the wedge 90° around a vertical axis, the cross-wind component of slope could be measured.

![Diagram of wind and water tunnel for measurements of slopes of waves generated by wind.](image)

There are a number of features of this technique which make it less than ideal. The most serious fault is that the brightness registered by the phototube is affected to some extent by the elevation of the water surface above the wedge; for example, as the crest of a large wave comes into the field of view, the rate of change of brightness with respect to slope is increased above its mean value because rays traced back from the water surface have a longer than average distance to travel between wedge and water surface. Thus the effect is a change of scale between slopes measured near the crest of waves and near the troughs. It can be shown that the change of scale is to the mean scale
as the change of depth of water is to the mean depth from water surface to wedge. At the highest wind speeds (12 m sec\(^{-1}\)) one may estimate the changes of scale as ±20% of the mean scale; at lower wind speeds the changes become rapidly smaller so that at 8 m sec\(^{-1}\) the changes of scale are estimated at ±5%.

Second, for angles of slope greater in magnitude than 40°, the brightness is not linearly related to slope. For angles less than 40° the maximum departure from linearity is found to be about 5%.

Third, the brightness depends slightly on cross-wind components of slope as well as on the intended (up/down-wind) component. This effect is expected to be negligible compared to the larger errors mentioned above.

A final difficulty is brought about by scattered light which enters the photocell and causes spurious signals (generally of lower frequency than the intended signal); this is reduced by painting the interior of the photocell telescope tubes dull black.

The wind speed in the tank was measured by a cup anemometer in the entrance nozzle. It was calibrated by comparison with a propeller vane anemometer with a National Bureau of Standards calibration. The position of the anemometer in a section where the air flow is uniform ensures that it measures the mean flow through the tunnel.

Measurements of air temperature, wet bulb temperature, and water temperature were made adjacent to the anemometer by mercury-in-glass thermometers.

Surface tension measurements in situ were made with a De Noury ring tensiometer only when the air blower was not operating. Some attention was paid to the question of surface films on the water, since these are known to affect wave generation markedly (see Keulegan, 1951; Van Dorn, 1953; Cox and Munk, 1954). It was found that, by continuous draining through an overflow trough at the lee end of the tank, the surface could be kept clean. This was confirmed by measurements of surface tension. Without draining, the surface film would be compressed to the lee end of the tank by strong winds; when the wind ceased it would then expand rapidly in a few seconds over the entire tank. With continuous draining, the process of reformation of the surface film took several minutes.

2.1 Wave Length Response of System. The finite dimensions of the spot on the water through which all rays pass on their way to the photocell form a "low-pass" filter to the signal. Slopes due to waves of length much less than the diameter of the spot are not effective in causing variations of brightness on the photocell; conversely, slopes
due to waves much longer than the diameter of the spot are properly recorded.

The size of the spot on the water is determined by three factors: (1) size of pinhole in telescope divided by magnification ratio between object and image; (2) degree of malfocussing (caused by rise and fall of waves); (3) perfection of lens in telescope. These factors were estimated separately. For observations of mean square slope (Section 3.1), the effective spot size was 2.0 mm in diameter. An improved lens system was used for observations of the frequency spectrum of slopes; the effective spot diameter was 0.7 mm.

Suppose that the spot radius is $r_0$ and that all light transmitted through the spot contributes equally to the signal. Suppose further that a wave of slope $s = s_0 \cos (kx - \omega t)$ crosses the field of view centered at $r = x \cos \theta = 0$ in an arbitrary direction $x$ and that the brightness is proportional to $s$ plus a constant. Then the photocell current is proportional to a constant plus

$$
\int_0^{r_0} \int_0^\infty s_0 \cos (kr \cos \theta - \omega t) \, r \, dr \, d\theta = 2\pi \int_0^{r_0} s_0 r J_0(kr) \, dr \, \cos \omega t
$$

$$
= \text{const} \, s_0 \frac{J_1(kr_0)}{kr_0} \cos \omega t,
$$

where $J_0$ and $J_1$ are Bessel's functions in standard notation. The mean square output is therefore proportional to $2 J_1(kr_0)/kr_0$.

This quantity lies between 1.00 and 0.90 for wave lengths $\lambda = 2\pi/k > 6.8 \, r_0$. For $\lambda = 1.7 \, r_0$ the output vanishes, and for smaller wave lengths the output remains always below 0.017. For values $2r_0 = 2.0, 0.7$ mm as above, the wave length of 90% response is $\lambda = 6.8, 2.4$ mm. These correspond (according to the usual theory of infinitesimal waves on still water) to frequencies of 42, 190 cps.

2.2 Calibration. A rectangular block of lucite supported on a horizontal axle, with its upper surface just above water level, is used to calibrate the slope-measuring system. When the block is rotated about its axle, the phototube registers a change of light intensity as though the water surface were tilted. In practice the lucite block is oscillated by a motor through a 24-degree arc and the phototube output is recorded.

2.3 Measurements of Phase Velocities. Here two photocell pickups are used. One is fixed while the other is mounted on a movable carriage so that the distance, $\xi$ (Fig. 1), which separates the two fields of
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<td>2.28</td>
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view can be varied. The electrical output signals are sent first through two narrow-band electrical filters tuned as nearly as possible to the same frequency and then to a correlator which measures the mean product of the filtered signals. The correlation shows a nearly sinusoidal variation as $\xi$ is varied. The wavelength of the sinusoidal variation is the apparent wavelength of the water waves having the same frequency as the filters. The apparent wavelength needs correction for the fact that the direction of propagation of the waves is not always parallel to $\xi$. The corrected wavelength multiplied by the frequency of the filtered disturbance yields the phase velocity.

3. LABORATORY MEASUREMENTS

3.1 Mean Square Slope. Mean square slopes were measured by passing the electrical output from a single photocell telescope to a squaring and averaging circuit. Data, recorded for three fetches (Fig. 2; Table I), show that below a certain wind speed, $U_0$, the mean square slope is extremely small; above this speed, however, the mean square slope increases very rapidly with wind speed at first, then more slowly, and finally more rapidly again. The magnitude of $U_0$, defined by extrapolating the mean square slope curve linearly to the abscissa, varies with fetch as shown in Table II.

<table>
<thead>
<tr>
<th>Fetch (m)</th>
<th>$U_0$ (m sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.47</td>
<td>2.9</td>
</tr>
<tr>
<td>2.14</td>
<td>2.5</td>
</tr>
<tr>
<td>3.20</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Using data from the three fetches, one can get a rough idea of the change of mean square slope with fetch. In Fig. 3 the data shown as circled points have been found from intersections of the freehand curves in Fig. 2 with lines of constant wind velocity. Although these data are sparse, they suggest that there is a sharply defined critical fetch for winds in the range 2.4 to 3.2 m sec$^{-1}$, above which the mean square slope increases rapidly from a very low value.

An attempt was made to find a variation of mean square slope with changes of air-water temperature difference. The tank water temperature was varied by heating and cooling while the air temperature was uncontrolled. The data are summarized in Table III. Here $\langle s^2 \rangle$ is reported relative to its value under "normal" conditions (group 2 in Table III). All measurements were made at a fetch of 2.14 m.
Figure 2. Mean square value ($s_{u^2}$) of up/down-wind slope component as a function of wind speed. The three curves refer to fetches of 3.20 m (upper curve), 2.14 m (middle), and 1.47 m (lower). The curves have been separated vertically for clarity.

Figure 3. Relation of mean square water surface slopes to fetch for various wind speeds, $U$. The circled points have been estimated by reading off selected values of the smooth curves shown in Fig 2. The dotted lines are extrapolated.
From these observations no consistent change of mean square slope with water temperature is apparent.

3.2 Frequency Spectrum of Slope. The signal at 2.14 m fetch from a single phototube telescope, recorded on magnetic tape and played back at a speed 37.5 times higher than the recorded speed, was analyzed on a spectrum analyzer of the analogue type (Fig. 4). Calibration of the analyzer was carried out by impressing on the same tape a "white noise" (i.e., a signal of uniform spectral intensity) produced by the shot effect in a multiplier phototube exposed to steady light. By comparing the spectrum of the white noise with sine-wave signals passed through the same system, it was found that the multiplier phototube output yielded a uniform spectral intensity (within the probable error of measurement, ± 0.5 db) from high frequencies down to 1 cps. Schwantes et al. (1956) have demonstrated with greater precision that some multiplier phototubes yield a uniform noise intensity above 1 cps. Calibration in terms of real units of slope was performed as indicated in Section 2.2.

The frequency analyzer consists of 30 band-pass filters spaced one-third octave apart (frequency ratio between bands = 1.23). The effective width of each filter band is also one-third octave. Each datum reported by the analyzer is then proportional to the total spectral intensity integrated over the band passed by one filter.

The input to the spectrum analyzer is composed of two parts: (1) a noise spectrum compounded of multiplier phototube shot noise and stray electrical pickup (mostly 60 cps noise introduced in the analyzer which shows up as a peak at 60/37.5 cps = 1.7 cps in Fig. 4) and (2)
signal proportional to the spectrum of $s_x$. The shot noise is “white” and hence can easily be removed if its intensity is known at any frequency. The observations at low wind speed (3.2 m sec$^{-1}$) show a high-frequency tail from 170 to 540 cps of uniform intensity. It is assumed that this is the photomultiplier noise and that the same noise

Figure 4. Slope spectrum times frequency, $fS(f)$, of up- and down-wind slopes as a function of frequency, $f$, for four wind speeds $U$ as shown. The fetch was 2.14 m.
level occurs for each of the spectra at all frequencies; accordingly, this source of noise has been subtracted from all spectra shown in Fig. 4. Some features of doubtful authenticity still remain. The rise of spectral intensity shown by the three lower curves beyond 300 cps is probably not real. It may be caused by imperfect frequency calibration of the magnetic tape at frequencies where large corrections are necessary or by nonlinear recording of slopes larger than tangent 40°.

The spectral intensity, $S$, is the contribution to mean square slope by waves in a unit frequency band:

$$\langle s_x^2 \rangle = \int_0^\infty S(f) \, df.$$  \hspace{1cm} (1)

Since $s_x$ is dimensionless, the dimensions of $S(f)$ are (cps)$^{-1}$.

The observations show that $S(f)$ tends to decrease with increasing frequency. Accordingly, it is clearer to plot $f \, S(f)$, since this quantity gives equal importance to high- and low-frequency waves (Fig. 4). By transforming eq. (1) to the form

$$\langle s_x^2 \rangle = \int_0^\infty f \, S(f) \, d(\ln f),$$

one sees that $f \, S(f)$ is the spectral intensity referred to the relative frequency band $\delta(\ln f) = \delta f/f$.

As a check on the spectra, mean square slopes calculated by integrating each spectrum are compared below with those observed directly (Section 3.1):

<table>
<thead>
<tr>
<th>Wind speed (m sec$^{-1}$)</th>
<th>Mean square slope from spectrum</th>
<th>Mean square slope observed directly</th>
</tr>
</thead>
<tbody>
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<td>3.2</td>
<td>$1.32 \times 10^{-2}$</td>
<td>$1.36 \times 10^{-2}$</td>
</tr>
<tr>
<td>6.1</td>
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<tr>
<td>12.0</td>
<td>12.6</td>
<td>8.26</td>
</tr>
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</table>

The agreement is satisfactory except at the highest wind speed. The causes of the poor agreement here are not entirely clear. Part of the difference can be accounted for by consideration of the poor lens system (Section 2.1) used to measure mean square slope, but this can hardly account for the whole effect.

3.3 Phase Velocity. Phase velocities were measured for three frequencies of waves: 6.6, 37, and 104 cps by the method outlined in Section 2.3. The data are summarized in Table IV. In columns 5–8, $f_0$ and $Q$ are respectively the center frequencies and quality fac-
tors of the filters. The effective \( f_0 \) (column 9) is intermediate between \( f_0 \) for each filter and is used together with the corrected wave lengths to compute the phase velocity. Estimations of apparent wave length, \( \lambda \), and the remaining quantities are described below. Corrections to the apparent wave length (column 11) have to be made for the fact that all wave velocities are not parallel to the axis of the tank. If all wave directions are within a small angular spread of the axis of the tank, it can be shown that the true wave length, \( \lambda_0 \) (column 16), is related to the apparent wave length, \( \lambda \), by a reduction factor \( F \), (column 15) according to

\[
\lambda_0 = \lambda F,
\]

where

\[
F = 1 - \frac{1}{2} \left( \frac{\sigma_y^2}{\sigma_x^2} \right).
\]

Here \( \sigma_x \) and \( \sigma_y \) are rms filter outputs when the wedge (Fig. 1) is oriented to respond to slopes up-down wind and cross wind, respectively.

If the angular spread of wave directions, \( 2\theta_0 \), is appreciable compared to

\[
\left( \frac{10 \lambda_0}{\pi \xi_m^2} \right)^{1/3},
\]

where \( \xi_m \) is the maximum separation of the photocell telescopes, then the foregoing method is no longer accurate. It is now necessary to know not only the value of \( \sigma_y^2/\sigma_x^2 \) but also the directional distribution

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<th>No. 2</th>
<th>Effective</th>
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<td></td>
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<td>( Q )</td>
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</tr>
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of the waves. Since results do not depend critically on this distribution, it has been assumed in the subsequent analysis that the directional spectrum of wave elevations is fan-shaped, i.e., the spectral intensity (at the frequency of observation) is uniform for wave directions less than \( \theta_0 \) (measured with respect to the axis of the tank) and is zero for wave directions larger than \( \theta_0 \). With this assumption it can be shown that the correlation between two pickups separated by a distance \( \xi \) will be proportional to

\[
G(\xi) [A \cos x + B \sin x],
\]

where

\[
x = \frac{2\pi \xi}{\lambda_0}; \quad A = \sqrt{\frac{\pi}{x}} C(\theta_0 \sqrt{\frac{x}{\pi}}) + \sqrt{\frac{\pi}{x^3}} S(\theta_0 \sqrt{\frac{x}{\pi}}) - \frac{\theta_0}{x} \sin \left( \frac{x \theta_0^2}{2} \right);
\]

\[
B = \sqrt{\frac{\pi}{x}} S(\theta_0 \sqrt{\frac{x}{\pi}}) - \sqrt{\frac{\pi}{x^3}} C(\theta_0 \sqrt{\frac{x}{\pi}}) + \frac{\theta_0}{x} \cos \left( \frac{x \theta_0^2}{2} \right);
\]

\( C, S \) are Fresnel’s integrals, \( C(u) + iS(u) = \int_0^u \exp \left( \frac{\sqrt{2}i\pi t^2}{2} \right) dt \); and \( G \) is a slowly varying factor which depends on the \( Q \) of the filters and on the accuracy with which one can associate a single wave length with waves of one frequency. In deriving the formula, the assumption has been made that \( \theta_0^2 \ll 1 \).

<table>
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<th>( N )</th>
<th>( N_{1/4} )</th>
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<th>( F )</th>
<th>( \lambda_0 ) (cm)</th>
<th>( c ) (cm sec(^{-1}))</th>
<th>( c_0 ) (cm sec(^{-1}))</th>
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Figure 5. The theoretical phase velocity, $c_\phi$, and frequency, $f$, for waves on still water in the absence of wind. The observational data are indicated by points which correspond to the wind speeds shown.
The difference between observed and theoretical phase velocities as a function of wind speed. The three graphs refer to three frequencies (in cps) of waves as noted on the right.

The half-beam width, $\theta_0$, can be found from $\sigma_y/\sigma_x$ according to a formula derived by Cox and Munk (1954).

In finding the apparent wave length, $\lambda$, the procedure has been to find the successive phototube separations, $\xi_n$, at which the correlation reverses sign. From a plot of $\xi_n$ vs $n$, a least square analysis yields $\lambda$ as well as an estimate of its standard error (see Table IV). Similar plots were made from the theoretical eq. (4) for suitable values of $\theta_0$. From a least square analysis of these plots, one gets a factor $F$ which can be used (eq. 2) to correct the apparent wave length.

Entered in columns 12 and 13 are the total number, $N$, of changes in
sign of the correlation as well as an estimate of the ordinal number, $N_{1/3}$, of the zero crossing (reckoned from zero separation) at which the amplitude of the correlation curve has dropped to one third of its initial value. $N_{1/3}$ permits an estimate of the factor $G(\xi)$, eq. (4), a discussion of which is postponed until Section 4.

The half-beam width, $\theta_0$ (column 14), is estimated together with its standard error from a measurement of $\sigma_y/\sigma_x$ (data not shown). Then, from the quantities $N$ and $\lambda$, one can estimate $\xi_m$ by $\xi_m \approx \lambda N/2$ and evaluate the criterion for the applicability of eq. (3). Using eq. (3) or (4), whichever is suitable, one finds $F$ (column 15) together with its standard error. Fortunately the correction factor is always close to unity and its uncertainty is generally of negligible importance for the corrected wave length, $\lambda_0$ (column 16). In column 17, $c$ is the phase velocity (and standard error) computed from $c = \lambda_0$ times (effective $f_0$). In column 18 $c_0$ is the theoretical phase velocity for free waves on still water without wind (Lamb, 1945: 459, eq. 2). Comparison of theoretical and observed quantities is shown in Fig. 5. In computing $c_0$ it has been assumed that the surface tension corresponded to the temperature measured by the wet bulb thermometer. Because of the uncertainty of this estimate, there is an uncertainty in $c_0$. No measurements of wet bulb temperature were made on 24-31 October, and here the temperature is estimated at 16 ± 4°C.

The quantity $c - c_0$ (column 19) shows the difference between the observed phase velocity and $c_0$. The same quantity is plotted in Fig. 6.

4. DISCUSSION

Measurements of mean square slope may be compared with measurements made in the open sea (Cox and Munk, 1954). Measurements of wind speed in the tank represent average speed throughout the air channel; it may be estimated that the level at which this speed is reached in the boundary layer next to the water is 4 to 8 cm above mean water level. Hence, oceanic measurements of wind speeds made at a height of 12.5 m must be reduced by a large and uncertain correction factor before a useful comparison can be made. I assume the factor is

$$\ln \left( \frac{6.0}{0.1} \right) / \frac{1.25 \times 10^4}{0.1} = 2.2 ;$$

this assumes (1) a logarithmic velocity profile near the water in both tank and ocean, (2) an effective height of wind observation in the tank of 6 cm, and (3) a roughness length in both the tank and ocean of 0.1 cm. Ocean measurements reduced in this way show
a linear increase of mean square of the up-down wind component of slope with wind speed from zero at no wind to a value of .044 at a wind speed of 6.3 m sec\(^{-1}\). Tank measurements showed that a mean square slope of .044 was reached at wind speeds of 6.4, 6.1, and 6.0 m sec\(^{-1}\) at fetches of 1.5, 2.1, and 3.2 m respectively. With due regard to the uncertainty of comparing wind speeds between tank and ocean, it appears that the mean square slope at 6 m sec\(^{-1}\) wind speed will not markedly increase beyond a fetch of 3 m. This is confirmed by the shape of the curves showing mean square slope as a function of fetch (Fig. 3). It appears that, regardless of the mechanism responsible for the explosive growth of waves just beyond the critical fetch, nonlinear effects rapidly set in to limit the slopes at fetches slightly greater than 3 m. On the other hand, it is clear that, for wind speeds lower than 6 m sec\(^{-1}\), oceanic measurements showed appreciable mean square slopes at wind speeds below the critical speed for the tank. This fact indicates that the critical wind speed decreases to a low value for the long fetches encountered in the open sea.\(^2\)

Curves showing the growth of mean square slope with fetch (Fig. 3) may shed some light on the question of the initial generation of waves. According to Jeffreys’ (1924–25) “sheltering” theory, the wave amplitude should, in the initial stages, increase exponentially down wind. If this were indeed the case, then observations of very small mean square slope at the critical fetch and rapid growth down-wind imply that waves must be extremely small at the up-wind end of the tank. This was not in fact the case. On viewing objects reflected in the surface, it appeared that there was more wave motion here than at a point just short of the critical fetch (but still so small as to be unmeasurable by the slope apparatus). These small waves at short fetches were probably caused by vibration of the tank and by fluctuating pressures set up by turbulence in the air stream as it flowed around the anemometer at the mouth of the wind channel and over a 1- or 2-cm lip (designed to keep waves from splashing water out at the up-wind end of the water channel). One concludes that there is not an exponential growth of waves at short fetches.

In contrast to Jeffreys’ development, in which the air pressure fluctuations are coupled to the already existing waves, Phillips (1957) has developed a theory of wave generation based on a model in which the pressure fluctuations are completely independent of the waves. The theory has been worked out for unlimited fetches and finite duration of winds only. However, Phillips makes the remark that

\(^2\) The waves generated at these low wind speeds are probably gravity waves; cf Fig. 2 in “Comments on Dr. Phillip’s Paper” (J. Mar. Res., 16(3): 241–245).
waves generated by steady winds at a small fetch are likely to be similar to those described by his theory at a short time after the sudden onset of wind. If this correspondence is valid, Phillips' calculation of the directional distribution of the waves should be comparable to wave measurements in the tank. According to Phillips' result, practically all of the waves of frequency \( f \) and wavelength \( \lambda \) will make an angle \( \theta_1 \) with the wind where

\[
| \theta_1 | = \cos^{-1} \frac{c(f)}{U_\lambda},
\]

(5)

c is the phase velocity of the waves, and \( U_\lambda \) is the "convection velocity" of the turbulent pressure fluctuations: that is, the average speed at which pressure fluctuations of wave length \( \lambda \) are drawn over the water. It seems plausible to assume, with Phillips, that \( U_\lambda \) is the mean wind velocity at a height \( \lambda/2\pi \) above mean water level.

This result contrasts with the assumption made in the analysis of phase velocities (Section 3.3) that waves were found at every angle to the wind within the fan-shaped beam \( | \theta | \leq \theta_0 \). Measurements of the ratio of mean square filtered slope components, \( \sigma_y/\sigma_x \), cannot distinguish between the two directional patterns of waves; but they do permit a check of eq. (5). In Table V, for waves at \( f = 6.6 \) cps, estimates of \( \theta_1 \) from eq. (5) are compared with estimates based on the measurement of \( \sigma_y/\sigma_x \) according to \( \tan \theta_1 = \sigma_y/\sigma_x \).

<table>
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<th>Mean wind speed ((cm \text{ sec}^{-1}))</th>
<th>( \lambda_0 ) ((cm))</th>
<th>( c ) ((cm \text{ sec}^{-1}))</th>
<th>( U_\lambda ) ((cm \text{ sec}^{-1}))</th>
<th>( \theta_1 )(Eq. 5)</th>
<th>( \theta_1 = \tan^{-1}(\sigma_y/\sigma_x) )</th>
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<td>648, 194</td>
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<td>390</td>
<td>4.60</td>
<td>30</td>
<td>286, 86</td>
<td>84°, 69°</td>
<td>39° ± 3°</td>
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</tbody>
</table>

The first three columns of Table V are taken from Table IV. In column 4, the two values of \( U_\lambda \) have been estimated by calculating the mean wind speed at a height \( 2\pi/\lambda_0 \) above the water, assuming a logarithmic velocity profile near the water, with a roughness length of \( \frac{1}{3} \) mm and 3 mm respectively. These are representative limits within which the value of \( \varepsilon_0 \) is likely to lie according to measurements in the open sea (Hay, 1955). The two values of \( U_\lambda \) so derived lead to two values of \( \theta_1 \) (eq. 5) shown in column 5. The value derived from observation of \( \sigma_y/\sigma_x \) is shown in column 6.
The uncertainty in $U_\lambda$ is very large; consequently, one must regard the values of $\theta_1$ calculated by eq. (5) as rough estimates. Nevertheless, it is difficult to see how the observed values of $\sigma_y/\sigma_x$ can be reconciled with eq. (5). It is concluded that waves do not run at the angle $\theta_1$ to the wind shown by eq. (5), and this suggests that the hypothesis adopted by Phillips is not correct for the initial generation of waves.

Direct cathode-ray oscillograph recordings of slope at 2.14 m fetch are shown in Figs. 7–9. In Fig. 7, one sees clearly that for each wind speed there are two preferred frequency bands of activity; this

![Figure 7](image)

**Figure 7.** Records of up- and down-wind slope component, $\varepsilon_y$, as a function of time for various wind speeds in m sec$^{-1}$ as follows: (a) 3.0; (b) 4.8; (c) 6.1; (d) 8.5; (e) 9.5; and (f) 11.5. (A record at 7.1 m sec$^{-1}$ is shown in Fig. 7b). The slope scale (vertical) is shown by three horizontal dashes to the right of each trace. The separation between dashes is 0.50 in slope. An upward deflection of the trace refers to a water surface element whose up-wind edge is higher than the down-wind edge. The time scale (horizontal) is shown at the bottom of the figure by large marks at one-second intervals and small marks indicating tenths of seconds. Time increases to the right.

![Figure 8](image)

**Figure 8.** (a) Initial generation of waves; (b) waves in equilibrium with the wind; (c) decay of waves after blower is turned off. These records show up- and down-wind slope (vertical) vs. time (horizontal) as in Fig. 6. The wind speed and fetch are 7.1 m sec$^{-1}$ and 2.14 m, respectively.
is shown also by the frequency spectra (Fig. 4). As the wind speed increases, the low-frequency band tends to low frequencies while the high-frequency band broadens and extends into high frequencies. The dip in the spectrum at low wind speed between the two bands occurs at the frequency of minimum-phase velocity (Figs. 4, 5). Consequently the changes of spectra are such as to keep both bands of activity moving at a speed proportional to the wind.

For winds greater than about 7 m sec\(^{-1}\) a new effect becomes important: high-frequency ripples become associated with the leading slopes of low-frequency wavelets, i.e., the region down wind of a crest but up wind of the preceding trough (Figs. 7d, e, f; 8b).

Munk (1955) has given two alternative explanations for similar observations in the ocean. According to the first, it is stated that the only stable position for ripples is on the leading slope of a wavelet because the orbital velocity of the wavelet creates a zone of convergence which sweeps ripples onto the leading slope. Some computations of the relative positions of ripples of various length to be expected with this hypothesis are given by Munk. It is thought that these computations are in error because they are based on equating ripple phase velocity to the combination of phase and orbital velocity of the underlying wavelet. Since the energy moves with the group velocity, it is felt that the correct computation should equate the ripple group velocity to the combination of phase and orbital velocity of the wavelet. Qualitatively both computations lead to the same result: that the short ripples should precede the long ones.

Munk's second explanation is based on intermittent generation of ripples by a moving pressure disturbance of short duration. This leads (according to a generalization of the well known solution of the "fish-line" problem) to a dispersive ripple train in front of the moving pressure. Munk associates the pressure disturbance with some (unknown) process located near the crest of wavelets.

From the rather irregular records shown in Figs. 7 and 8, it seems impossible to deduce which of these processes is operative. Accordingly some records were made of wavelets artificially generated by a plunger (Fig. 9). These also show the characteristic ripple trains on the leading slopes of wavelets even though the plunger supplied energy only at the wavelet frequency. It may be that ripples were generated because there was a finite spread of direction of the wavelets; where two crests intersected, there was a momentary nonlinear interaction which gave rise to the ripples.

The rather regular appearance of ripples on the records of Fig. 9 permits not only a rough estimate of the period of individual ripples but comparison with Munk's hypotheses as well. The result is that
plots of position of ripple vs ripple frequency are consistent with Munk's first hypothesis (as modified for equality of ripple group velocity to wavelet phase plus orbital velocity).

It seems likely then that the energy of the ripples is trapped on the leading slope of wavelets by convergence due to the distribution of the orbital velocity in that position. The possibility of trapping in this manner requires that the magnitude of the difference between the phase velocity of the wavelets and the group velocity of the ripples be not greater than the maximum orbital velocity of the wavelets. To a considerable extent this possibility is, therefore, an artifact of the wave tank used in these experiments, because the fetch is sufficient to allow the generation of wavelets with large orbital velocities but is not long enough to permit development of wavelets of high phase velocity. In the open sea, where a sharp spectral peak of low-velocity wavelets is not to be expected, one expects that trapping of ripples will not be such a noticeable and persistent phenomenon. Some evidence on this point is shown in Fig. 14; in the left-hand photograph only a few ripples appear to be trapped on the leading (top) slopes of wavelets about 50 cm long; in the right-hand photograph the whole surface appears uniformly covered with ripples.

Also shown in Fig. 9 are records of plunger-generated waves as modified by winds (Fig. 9b, c, d). The effects of wind appear to be an increase in the average energy and frequency of ripples. The increase of energy may be due to generation by wind. The change of frequency suggests that the relation between frequency and group velocity of ripples is altered by the effects of wind in such a way that only high frequency ripples can keep up with the wavelets.

Measurements show pronounced changes of phase velocity (Sec-
tion 3.3) with wind speed. There are four factors which are of the appropriate order of magnitude to be of importance. They are (1) finite amplitude of waves, (2) effects of orbital velocity of low-frequency wavelets, (3) water currents, and (4) dynamic effects of wind.

The theory of finite amplitude capillary-gravity waves has been initiated by Sekerzh-Zenkovitch (1956). The velocity, \( c \), of waves of constant form is given in the following form to the third order:

\[
c^2 = c_0^2 + \Delta(\lambda, \epsilon^2);
\]

here \( c_0 \) is the velocity of infinitesimal waves, \( \Delta \) is a function of wave length \( \lambda \) and \( \epsilon \), a parameter (which tends in the limit of infinitesimal amplitude to the maximum slope of the wave). The equation is equivalent to the following expression, correct to the third order in \( \epsilon \):

\[
c = c_0 \left( 1 + \frac{1}{2} \frac{\Delta}{c_0^2} \right).
\]

Figure 10. Effects of finite amplitude on wave velocity as a function of wave length, \( \lambda \). The quantity \( \frac{1}{2} \frac{\Delta}{c_0^2} \epsilon^2 \) is defined in eq. (5), Section 4. The curve is not valid at the wave length indicated by the arrow.
The quantity $\frac{1}{2}[\Delta/(c_0^2 \epsilon^2)]$ is shown in Fig. 10. Certain assumptions in the derivation make the solution invalid when the wave length is such that $n$ capillary waves have the same length as one gravity wave. The wave length where $n = 2$ is indicated in the figure by an arrow.

\[ \frac{1}{2}[\Delta/(c_0^2 \epsilon^2)] \]

**Figure 11.** Comparison of plunger-generated wave (solid line) (see Fig. 8a) with slope profile predicted by finite amplitude theory of Sekerzh-Zenkovich (dashed line).

For $\lambda > 3$ cm, the value of $\frac{1}{2}[\Delta/(c_0^2 \epsilon^2)]$ is positive and tends to the limit $\frac{1}{2}$ as $\lambda \to \infty$. Hence the phase velocity approaches $c_0(1 + \frac{1}{2}\epsilon^2)$. It is clear that waves of length $\lambda > 3$ cm behave substantially like finite amplitude gravity waves according to Stokes' solution. On the other hand, if $\lambda < 1$ cm, the effect of finite amplitude on the phase velocity is markedly reduced and reversed in sign.

Measurements at 6.6 cps ($\lambda = 4.7$ cm, $c_0 = 28.9$ cm sec$^{-1}$, wind speeds from 3.9 to 8.8 m sec$^{-1}$) give $c/c_0 = 1.075 \pm .010$ (Table III). Taking $\frac{1}{2}[\Delta/(c_0^2 \epsilon^2)] = 0.62$ from Fig. 10 and $\frac{1}{2}\Delta/c_0^2 = c/c_0 - 1 = .075 \pm .010$, one finds $\epsilon = .35 \pm .05$. To find the value of maximum slope to which this value of $\epsilon$ corresponds, it is necessary to fit some wave profiles to the Sekerzh-Zenkovich wave profile formula. One wave from Fig. 9a has been so fitted (Fig. 11). The best fit was for
\( \epsilon = 0.33, \lambda = 7.0 \text{ cm}. \) (The wiggles on the computed profile are probably the result of pushing the third order theory beyond its limits of validity.) The maximum slope of the observed wave was about 0.42. One estimates then that wavelets with maximum slopes of 0.42 will have a value of \( \epsilon \) sufficiently large to account for the observed increase of phase velocity. But this is a fairly representative slope for waves near 6.6 cps at the wind speeds where phase velocities were measured. In conclusion, the increase of phase velocity at 6.6 cps for moderate winds is probably the effect of finite amplitude.

\[ \text{Figure 12. Correlogram of slopes measured at two points a distance } \xi \text{ apart. Measurements from } 28 \text{ November;} f_s = 104 \text{ cps, wind speed } = 7.3 \text{ m sec}^{-1}. \]

Phase velocities of 37 and 104 cps waves will scarcely be affected by their finite amplitude according to Sekerzh-Zenkovitch's theory, but it can be shown that the other three factors will be effective. Unfortunately it does not now appear possible to disentangle the effects.

Measurements of phase velocity gave as a by-product an estimation of the degree to which one can associate a single wave length with a single frequency of wave. According to the usual theory of infinitesimal waves, those of a single frequency would have an accurately defined wavelength. The effect of waves running in a beam of directions on the correlogram of slopes measured at two points is to reduce, for large \( \xi \), the amplitude of the oscillations of the correlation rather slowly. This reduction is shown by factors \( A \) and \( B \) of eq. (4), Section 3.3. The effect is rather well illustrated by Fig. 12.

Most correlograms showed a much more rapid decrease of amplitude as \( \xi \) was increased, and the effect could not have been caused by the
directional beam of waves. An extreme example is shown in Fig. 13 where the decrease of amplitude was so great that it was not possible to measure an apparent wave length at all.

A variation in wave length for waves of a single frequency can be caused by variation of any of the factors which alter phase velocity or by processes of generation and decay. The latter can be illustrated by a simple model. Suppose the slope component due to a narrow range of frequencies of waves can be represented by

\[ s(x, t) = \sum_i g(x - x_i) f [x - x_i - v(t - t_i)] \cos[k(x - ct)], \]

where \( f(x) \) is a slowly varying function compared to \( \cos(kx) \) and \( g(x) \). The parameters \( x_i, t_i \) indicate the general location and epoch of a single train of waves, \( f(x - vt) \) represents the motion of the envelope at group velocity, \( v \), and \( g(x) \) represents processes of generation and decay. There are two extreme cases to consider: (1) the individual trains are generated completely at random so that \( x_i, t_i \) are independent and random variables; (2) the successive trains overlap in a regular fashion so that the sum represents a periodic function of time. In the first case the correlation of slopes at two points a distance \( \xi \) apart becomes

\[ < s(x, t) s(x + \xi, t) > = G(\xi) \cos(k\xi), \]

where

\[ G(\xi) = \int_{-\infty}^{\infty} g(x) g(x + \xi) \, dx \]

is a measure of the distance traversed by a single wave train; in the second case, the correlation is proportional to \( \cos k\xi \).

The first example is representative of random generation of waves while the second represents generation which is locked in phase with the already existing waves. To this extent they represent two theoretical models proposed respectively by Eckart (1953) and Phillips (1957) on the one hand and by Jeffreys (1924, 1925) on the other.

The measurements of \( N_{1/3} \) (Table IV; Section 3.3) can therefore yield some estimates of \( G(\xi) \) provided variations of phase velocity (see above) are negligible.

In the present data it is not possible to rule out the effects of variations of phase velocity. The values of \( N_{1/3} \) must then be regarded as lower limits on the distance (measured in wave lengths) of travel of a single randomly generated wave train. They do not yield any information on nonrandom generation processes.

Measurements showed that \( N_{1/3} \) was surprisingly small. In only four out of 15 measurements was \( N_{1/3} \) larger than three. The largest values are associated with low winds.
Figure 14. Photographs of the sea surface made from Scripps pier 29 March 1956. Wind speed was 6.5 m sec$^{-1}$ measured 12 m above sea level. The pronounced white dots on one photograph are reflections of the sun. The other photograph was exposed when the sun was covered with a cloud; the tiny white dots are reflections of a gas-discharge lamp supported 10 m above sea level. The scale indicates distances in meters. In these photographs the wind was blowing from an offshore direction, hence the fetch was almost unlimited. Some of the larger gravity wave motions shown may be due to reflections from piles supporting the pier, but it was clear to the eye that the ripples were driven by the wind towards the pier.
5. ACKNOWLEDGMENTS

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VAN DORN, W. G.
This stimulating paper by Dr. Cox (1958) provides a great deal of information on the inception of high frequency components of wind waves, on which few data of comparable extent have been obtained hitherto under controlled conditions. In these comments I will endeavour to show to what extent Dr. Cox's observations can be interpreted within the present theoretical framework and to what extent they indicate its limitations, so pointing to the need of further investigation.

The Role of Turbulence in Initiating Waves. In the resonance theory (Phillips, 1957, 1958b), pressure fluctuations on a water surface resulting from a turbulent air stream are considered as a means of raising waves. Under conditions of large fetch in all directions and of short duration, it is found that a whole spectrum of waves is generated, with directional maxima for components whose wavenumbers satisfy the resonance condition (Phillips, 1958c: eq. 2). Clearly, an a priori condition for the relevance of this theory requires that the air stream should contain pressure fluctuations having a length scale comparable with the wavelength of the waves excited. If this condition is not met (for example, if the air boundary layer is laminar), then any waves that might be observed would necessarily have to be described in terms of some other mechanism, such as that considered by the instability theory of Lock (1954). Even if the boundary layer is turbulent, there is no prior guarantee that the resonance mechanism will be dominant. For example, if the boundary layer thickness is less than, say, one or two centimeters, the pressure fluctuations have a length scale (of the order of half the boundary layer thickness, according to recent work—see Townsend, 1956) corresponding to the wavelength of capillary ripples, which are heavily damped by viscosity. However, there may still be an instability resulting in growth of waves of comparatively long wavelength, as Miles (1957) has shown. Presumably this was the type of wave observed by Jeffreys (1924, 1925) in his original experiments.

Turning to Dr. Cox's paper, one of his most interesting results is the observation of a relation between wind speed and the minimum fetch at which waves begin to grow. In view of the crucial dependence
of wave growth upon wind structure, it seems natural to inquire whether this relation might be associated with some property of the air boundary layer near the water. One’s first supposition is that this critical point might be associated with transition from laminar to turbulent flow in the boundary layer. Forming the Reynolds number \( R = Ux/\nu \), where \( U \) is the free stream speed, \( x \) the fetch, and \( \nu \) the kinematic viscosity of the air, the values of \( R \) corresponding to the critical fetches shown by Dr. Cox (1958: fig. 3) range from \( 3.2 \times 10^6 \) to \( 5 \times 10^5 \). It is known that, in the flow over a flat plate, transition to turbulence occurs at \( R \) values between \( 10^6 \) and \( 10^7 \) (Goldstein, 1938: 71), the lower values occurring when greater disturbances are present in the air stream. The boundary layer over the smooth initial part of the water surface differs in detail from that over a flat plate, but the transition Reynolds number would be expected to be of the same order of magnitude. The presence of the anemometer and the unavoidable presence of the small step at the leading edge of Dr. Cox’s tank (1958: fig. 1) would tend to bring about transition at Reynolds numbers low in the range \( 10^5 \) to \( 10^6 \).

It seems plausible therefore to associate this critical fetch observed by Dr. Cox with transition of the air boundary layer from laminar to turbulent flow. Shortly after transition occurs, we would expect waves to grow under the influence of the turbulent pressure fluctuations. The boundary layer thickness at this point is of the order of 5 cm, so that there certainly exist pressure fluctuations of a length scale of several centimetres. An explanation in these terms is also consistent with Dr. Cox’s remark that, for the long fetches encountered in the open sea, the critical wind speed decreases to a low value, since under such conditions the boundary layer Reynolds number is large indeed and the air motion invariably turbulent.

This explanation also enables us to make some quantitative predictions based on the ideas of the resonance theory. Comparison of these predictions with Dr. Cox’s observations is encouraging but not decisive. It has been shown (Phillips, 1958b) that, according to the resonance theory, the mean square slope should initially increase linearly with fetch from the point of inception. This is not inconsistent with Dr. Cox’s fig. 3, where the broken lines represent extrapolations. This linear relation would be expected to fail as soon as nonlinear interactions between the wave components become significant. The slope of this line cannot be estimated because of our lack of the right detailed information on the structure of the turbulent boundary layer, but the theory predicts that it should be approximately proportional to \( U^3 \). The circled points in his figure indicate a variation as \( U^{2.4} \), with a fairly high uncertainty in the exponent—
again close but not decisive. The variation of mean square slope with wind speed at constant fetch (Cox, 1958: fig. 2) is more difficult to express simply, and no definite prediction has been made.

**The Occurrence of Resonance Waves.** The resonance theory does predict, as Dr. Cox mentions in his §4, that there will be directional maxima in the wave field at angles $\theta$ to the wind (see Cox, 1958: eq. 5), provided the fetch is large in all directions. However, in a long narrow tank such as the one used in these experiments, when the wind velocity is large compared to the wave velocity, these waves travel almost across the tank. The fetch appropriate for these waves is only a little more than the tank width at most (or half the width if observations are made at the center), which is much less than that for waves travelling along the tank. Thus there is a strong weighting in favour of the latter, and it would be rather surprising if, under these conditions, true resonance waves made a significant contribution at all to the wave field. Apparently the waves observed here are still associated with turbulence but are not the true resonance waves for which this interaction is (for unlimited fetch) most efficient. Such wave components are also considered in the paper to which Dr. Cox here refers.

It seems, then, that the correspondence is not close and that any agreement between columns 5 and 6 of Table V in Dr. Cox’s paper would have been surprising.

**The Frequency Spectrum of Slope.** It is of interest to examine the possibility of making predictions concerning the shape of the frequency spectrum $s(f)$ of the surface slope under the saturation conditions of the equilibrium range (Phillips, 1958a). For gravity waves, the arguments I gave in that paper lead us to expect that, in the equilibrium range, $s(f)$ should be a function of $f$ and $g$, the gravitational acceleration alone, and independent of $U$ and fetch $x$ in particular. The slope frequency spectrum $s(f)$ has the dimension (time). On dimensional grounds, the only possibility is

$$s(f) \propto f^{-1}$$

$$f s(f) = \alpha_1, \text{ a constant}$$

provided $f$ is much less than the frequency associated with waves of minimum phase velocity or for water $f \ll 10 \text{ c.p.s.}$ This represents an upper limit on the function $s(f)$; the slope spectrum would be expected to fall below this limit if the energy supply is insufficient or if the dissipation is too large.
In the capillary range we are on less certain grounds, since viscosity may well be an important parameter as well as surface tension $T$. However, if the energy supply to the capillary ripples by nonlinear interaction or by direct action of the wind is sufficient to overcome the viscous dissipation, then an upper limit might be sought by neglecting viscosity and by seeking a saturation spectrum determined by $f$ and $T/\rho$ alone, where $\rho$ is the water density. On dimensional grounds, we find again that

$$f s(f) = \alpha_2, \text{ constant}, \quad (2)$$

but in this instance it is for frequencies where $f \gg 10$ c.p.s. for water. For extremely large values of $f$, (2) will inevitably fail, since the energy supply is limited and since the rate of viscous dissipation increases sharply for increasing $f$. The limit of validity of (2), then, is near a high frequency “cut off” $f_{max}$, which is higher when the energy transfer processes are more vigorous.

This idea of an equilibrium range for capillary ripples was advanced to me by Dr. Bruce Hicks at the University of Illinois, who pointed out in conversation that the surface displacement spectrum is of the form

$$\Phi(f) \propto \left(\frac{T}{\rho}\right)^{2/3} f^{-7/3},$$

subject to the same conditions as (2).

This type of dimensional argument cannot predict anything definite about the form of $s(f)$ in the range where both gravity and capillary effects are important, since we have an additional dimensionless ratio $Tf^3/\rho g^2$. A full investigation of the nonlinear interaction process would be required before the behaviour of $s(f)$ in this range could be predicted. Nor can such an argument say anything on the interesting details of the capillary patterns on the forward slope of larger waves described in Dr. Cox's paper.

Turning now to the results shown in Dr. Cox's fig. 4, we see many of the features described above. The function $f s(f)$ is approximately constant, even near $f = 10$ c.p.s. from the unsaturated low frequency end of the spectrum to the high frequency cut off. The magnitude of $f s(f)$ in this range seems independent of wind speed, as predicted by (1) and (2), and the high frequency cut off $f_{max}$ decreases at the lower wind speeds. At the lowest wind speed shown in Dr. Cox's fig. 4, the saturated range is quite short, and as he points out, it has a fairly definite dip near $f = 10$ c.p.s. If this is a real effect, it would bear some investigation.
It has been a privilege to comment on Dr. Cox's paper, and I hope that these remarks may encourage efforts to obtain a better understanding of these complex phenomena.

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