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THE EQUATORIAL CURRENTS OF THE EASTERN PACIFIC AS MAINTAINED BY THE STRESS OF THE WIND

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INTRODUCTION

Sverdrup (1947) has shown that the main features of the distribution of mass in the eastern equatorial Pacific can be derived exclusively from the average wind stress distribution. In this paper the method for computing the wind stress will be developed, the treatment of the oceanographic observations will be discussed, the field of mass and of mass transport as derived from the oceanographic and wind observations will be dealt with, and finally an analytic solution will be given.

Counter currents have long been recognized from ships' records in each of the equatorial oceans, and their detailed structures in the Atlantic and Pacific Oceans have been described by Sverdrup (1932) and Defant (1936).

Montgomery (1940) and Montgomery and Palmén (1940) regarded the counter current as a downslope flow in the region of the calm belt, retarded only by frictional forces at the lateral and underlying boundaries of the current. They assumed that the hydrostatic head necessary to balance the friction was maintained by a piling up of water in the west due to the traction of the adjacent trade winds.

Stockmann (1945) has shown that it is necessary only that a sufficient lateral variation exist in the velocity of a wind of constant direction, blowing over a closed homogeneous sea, in order to give rise to a steady surface "compensation current," directed opposite to the prevailing wind. He has extended this theory to include the equatorial counter current developed in a homogeneous ocean (1946a, 1946b). In subsequent papers he includes non homogeneous conditions.
Reid: The Equatorial Currents of the Eastern Pacific

(1946c), and takes lateral friction into account (1946c, 1946d), as suggested earlier by Rossby (1936a) and Montgomery (1940). In all his papers, Stockmann has neglected the fact that the Coriolis parameter varies with latitude.

Sverdrup (1947) takes into account the variation with latitude of the Coriolis parameter in his treatment of wind-driven currents in a baroclinic ocean, but he neglects lateral friction. His theoretical treatment, as well as that of Stockmann (1946c, d), assumes that it is possible to explain the predominant characteristics of the permanent equatorial currents and accompanying distribution of mass in a non-homogeneous sea by considering the wind stress as the only external factor explicitly involved. This would imply that the thermodynamics of the system can be ignored, and that only conservation of mass and of momentum need be considered. Such considerations evidently cannot completely account for the dynamics of the equatorial oceanic circulation, but it is intended here to show that a good approximation is achieved.

BASIC THEORY

Since we are dealing with a baroclinic medium wherein the pressure gradient generally vanishes at a moderate depth, it is apparent that horizontal motion and consequently frictional forces become negligible with depth. The net volume forces of the upper layer must therefore be balanced by the frictional stress exerted on the surface by the wind. The energy transferred to the water by the wind stress must be partly utilized by the work of cross isobaric flow and partly dissipated by turbulence in the upper layers if permanent currents are to be maintained.

Solutions in Terms of the Wind Stress. The principal assumptions, upon which Sverdrup (1947) bases his theoretical expressions, are as follows:

(a) there exists no acceleration,
(b) the fluid is free of lateral friction,
(c) the field of mass is in steady state,
(d) there exists a depth common to the whole fluid region at which horizontal and vertical motion is vanishingly small,
(e) vertical motion is absent at the surface, and
(f) the north–south component of wind stress is independent of longitude.

The above conditions lead to particular solutions of the equations of motion and continuity. The field of mass, which can be described in the horizontal by a function $P$, is given by the expressions
The function \( P = P (xy) \) is the vertical integral of pressure defined sufficiently by

\[
P = \int_0^d p \, dz + \text{constant},
\]

where \( p = p(xyz) \) is the pressure, \( z \) is the vertical coordinate measured downward from the surface, and \( d \) is the depth of no motion. In (1) \( x \) and \( y \) are the east and north coordinates, respectively, of a rectilinear system whose origin is situated on the equator; \( x_0 = x_0(y) \) is the position of the eastern boundary; and \( x_1 = x_1(y) \) is the locus of an arbitrary path west of \( x_0(y) \). The term \( R \) is the radius of the earth, \( \varphi \) is the latitude (positive north of the equator), \( \tau_z = \tau_z(xy) \) and \( \tau_y = \tau_y(xy) \) are the east and north components of wind stress, respectively, and \( \bar{\tau}_z' = \bar{\tau}_z'(y) \) is the average stress between the continent and the western path, \( x_1 \), i. e.,

\[
\bar{\tau}_z' = \frac{1}{(x_1 - x_0)} \int_{x_0(y)}^{x_1(y)} \tau_z dx.
\]

In (1b) \( \omega \) is the angular speed of the earth, and \( M_{z0} = M_z(x_0y) \) represents the eastward transport at the boundary.

The mass transport components are given by the expressions

\[
M_x = \frac{x_1 - x_0}{2\omega \cos \varphi} \left( \frac{\partial \bar{\tau}_x'}{\partial y} \tan \varphi + \frac{\partial^2 \bar{\tau}_x'}{\partial y^2} R \right) + M_x, \quad (4a)
\]

and

\[
M_y = -\frac{R}{2\omega \cos \varphi} \frac{\partial \tau_x}{\partial y}, \quad (4b)
\]

where \( M_x = M_x(xy) \) and \( M_y = M_y(xy) \) are the east and north components of the net mass transport of water per unit time through a column of depth \( d \) and unit widths. In terms of the density, \( \rho(xyz) \), and the velocity components, \( v_z(xyz) \) and \( v_y(xyz) \), the mass transport is given by the expressions

\[
M_z = \int_0^d \rho v_z \, dz, \quad M_y = \int_0^d \rho v_y \, dz.
\]

Sverdrup approximates the American west coast by a north-south
vertical boundary, in which case \( M_{x0} \) vanishes as required by the kinematic boundary condition (i.e., that the normal velocity at the boundary be zero). In general it can be shown, on the basis of (4b), that the transport for any vertical boundary takes the form

\[
M_{x0} = -\frac{R}{2\omega \cos \varphi} \left( \frac{\partial \tau_x}{\partial y} \right)_0 \tan \theta,
\]

where the zero subscript refers to the boundary, and \( \theta = \theta(y) \) is the bearing of the coast.

It can be demonstrated, however, that for the stress distribution of the trade wind area,

\[
M_{x0} = \frac{R}{\omega \cos \varphi} \left( \frac{\partial \tau_x}{\partial y} \right)_0 \tan \theta < \frac{\partial^2 \tau_x}{\partial y^2} R.
\]

Furthermore, the mean value of \( \tan \theta \) is approximately unity. This means that \( M_{x0} \) is negligible compared with \( M_z \) if \( (x_1 - x_0) \) is of the same order of magnitude as \( R \), and since \( \tan \varphi \) is less than unity, (1b) reduces approximately to

\[
\frac{\partial P}{\partial y} = - (x_1 - x_0) \frac{R}{\omega \cos \varphi} \frac{\partial^2 \tau_x}{\partial y^2} \tan \varphi + \tau_y \equiv f_2(\tau), \text{ for } |x_1 - x_0| > R. \quad (6)
\]

Likewise, (4a) becomes approximately

\[
M_z = \frac{(x_1 - x_0)}{2\omega \cos \varphi} R \frac{\partial^2 \tau_x}{\partial y^2} \equiv f_3(\tau), \text{ for } |x_1 - x_0| > R. \quad (7)
\]

Near the boundary the more exact equations, (1b) and (4a), must be used.

Equations (1a) and (4b) are averaged over the range from \( x_1(y) \) to a second arbitrary path, \( x_2(y) \), neglecting the boundary contributions to the derivative of mean stress\(^2\):

\[
\frac{\Delta P}{\Delta x} = - R \tan \varphi \frac{\partial \tau_x}{\partial y} + \tau_x \equiv f_1(\tau), \quad (8)
\]

and

\[
\bar{M}_y = - \frac{R}{2\omega \cos \varphi} \frac{\partial \tau_x}{\partial y} \equiv f_4(\tau). \quad (9)
\]

In (8) and (9) \( \bar{\tau}_x = \bar{\tau}_x(y) \) is the mean stress between the arbitrary boundaries, defined by an expression similar to (3), where \( x_0 \) is replaced

\(^2\) Terms of the form \( \frac{\tau_x(y) - \tau_x(x_1y)}{\Delta x} \frac{dx_1}{dy} \) or \( \frac{\tau_x(x_2y) - \tau_x(y)}{\Delta x} \frac{dx_2}{dy} \), thereby neglected, are of small consequence except for extreme variation of \( \tau(xy) \) with \( x \), or extreme values of \( dx_1/dy \) or \( dx_2/dy \).
by $x_2$; $P = P(x_1y) - P(x_2y)$ is the difference of $P$ for two stations at
the same latitude; and $\Delta x = (x_1 - x_2)$ is the range. In (9) $\bar{M}_v =
\bar{M}_v(y)$ is the mean north-south transport between $x_1$ and $x_2$.

The approximations (6), (7), (8), (9) are dependent upon the existence of but one solid boundary. Consequently, a second kinematic
boundary condition to the west cannot in general be satisfied by the
solutions, and application to only a limited region of the Pacific is in
order. This inadequacy of the solutions is presumably due to neglect
of field accelerations.

Solutions Based Mainly on Oceanographic Data. The pressure can
be determined to a very close degree of approximation by the hydro-
static equation,

$$dp = \rho dD,$$

where $D = D(xyz)$ is the geopotential depth. Since we are ultimately
concerned with differences of $P$, the anomaly of geopotential between
the isobaric surfaces $p(xyz)$ and $p(xyd)$ can be substituted for $D$.
Integrating (10) and making use of (2), the function $P$ takes the form

$$P = \bar{\rho} \int_0^d (\Delta D_d - \Delta D)dz + \text{constant},$$

where $\Delta D = \Delta D(xyz)$ is the geopotential anomaly between the
surfaces $p(xy0)$ and $p(xyz)$, and $\Delta D_d$ is that between the surfaces
$p(xy0)$ and $p(xyd)$. The term $\bar{\rho}$ is the mean density of the column
of depth $d$.

The mass transport can be obtained from the following expressions:

$$M_x = \frac{1}{2\omega \sin \varphi} \left( \tau_y - \frac{\partial P}{\partial y} \right),$$

(12a)

and

$$\bar{M}_v = \frac{-1}{2\omega \sin \varphi} \left( \bar{\tau}_x - \frac{\Delta P}{\Delta x} \right),$$

(12b)

obtained directly from the integrated equations of motion. These
equations are a good approximation for $|\varphi| > 0$. To avoid an inde-
determinate expression at $\varphi = 0$, equation (12a) is differentiatied
with respect to $y$ and solved for $M_x$ at $\varphi = 0$. This yields

$$M_x = \frac{R}{2\omega} \frac{\partial}{\partial y} \left[ \tau_y - \frac{\partial P}{\partial y} \right], \ \varphi = 0.$$

(13a)

Similarly, the north-south mass transport is

$$\bar{M}_v = -\frac{R}{2\omega} \frac{\partial}{\partial y} \left[ \bar{\tau}_x - \frac{\Delta P}{\Delta x} \right], \ \varphi = 0.$$

(13b)
If lateral friction and acceleration are not negligible in the vicinity of the equator, then equations (13a, b) are rigorous only if the former quantities are constant or attain an extreme value at $\varphi = 0$.

The above equations serve as an independent means of checking the expressions involving stress only.

**WIND STRESS THEORY**

The expression used for calculating the stress of the wind, $\tau (txy)$, is

\[
\tau (txy) = \rho' \nu^2 u^2,
\]

where $\rho'$ is the density of the air at sea level, $u = u (txy)$ is the wind speed at a fixed height, and $\nu^2$ is the resistance coefficient, a non-dimensional parameter. In general $\nu^2$ is a function of the roughness of the surface and of the height at which the wind speed is measured. Rossby and Montgomery (1935), Rossby (1936b), and Munk (1947) have shown that a discontinuity in roughness occurs at a wind force of about 4 Beaufort. Assuming that the estimated wind velocity applies to a height of 8 meters, the values of the resistance coefficient above and below the critical wind speed are approximately:

\[
\begin{align*}
\nu_1^2 &= 0.8 \times 10^{-3}, \quad u < \text{Beaufort 4}, \\
\nu_2^2 &= 2.6 \times 10^{-3}, \quad u > \text{Beaufort 4}.
\end{align*}
\]

These values, adopted by Sverdrup, et al. (1942), hold strictly for air of indifferent stability. It will be assumed here that the temperature lapse rate over the equatorial Pacific, on the average, equals that at indifferent equilibrium.

As indicated by Montgomery and Palmén (1940), because of the quadratic relationship between $\tau$ and $u$, the variability of the wind must be properly considered in evaluating the mean stress exerted by the wind. The complex variability of the wind velocity can be examined according to magnitude fluctuation and directional variation independently. The effect of variable magnitude is treated theoretically below.

The variability of wind speed in the *trade wind region* is assumed to be such that the distribution of magnitude about the mean, for any given direction and position, can be closely approximated by the Guassian law of error. At low mean speed the actual distribution probably becomes asymmetrical, which is contrary to the normal Guassian curve; but this fact is not important, since the stress exerted by a weak wind is exceedingly small.

In the following analysis, it is implied that the surface roughness is a
discontinuous function of the wind speed. The critical speed is taken
as 670 cm/sec.

For a Gaussian distribution, the time mean of the stress, \( \tau (xy) \), in
any specified direction, is given by the expression

\[
\tau (xy) = \frac{\rho' \int_{-\infty}^{\infty} \gamma^2 u^2 f \, ds}{\int_{-\infty}^{\infty} f \, ds}, \tag{15}
\]

where the frequency \( f \) and the parameter \( s \) are defined as follows:

\[
f = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{s^2}{2} \right), \tag{16a}
\]

and

\[
s = \frac{u - \bar{u}}{\sigma}. \tag{16b}
\]

In (16), \( \bar{u} = \bar{u} (xy) \) is the time mean of wind speed, and \( \sigma \) is the stand-
dard deviation of \( u \).

The denominator of (15) is unity, so that in the special case where
\( \bar{u} = 670 \text{ cm/sec} \), the equation takes the form

\[
\tau (xy) = \rho' \left[ \gamma_1^2 \int_{-\infty}^{\infty} u^2 f \, ds + \gamma_2^2 \int_{0}^{\infty} u^2 f \, ds \right]. \tag{17}
\]

Inserting the expressions for \( f \) and \( u \) in terms of \( s \), from (16), it follows
that

\[
\tau (xy) = \frac{\rho'}{\sqrt{2\pi}} \left\{ (\gamma_1^2 + \gamma_2^2) \left[ \sigma^2 \int_{0}^{\infty} s^2 \exp \left( -\frac{s^2}{2} \right) \, ds + \bar{u}^2 \int_{0}^{\infty} \exp \left( -\frac{s^2}{2} \right) \, ds \right] + (\gamma_2^2 - \gamma_1^2) 2\sigma \bar{u} \int_{0}^{\infty} s \exp \left( -\frac{s^2}{2} \right) \, ds \right\}. \tag{18}
\]

Upon evaluation of the definite integrals, (18) simplifies to

\[
\tau (xy) = \rho' \left[ \frac{\gamma_1^2 + \gamma_2^2}{2} \left( \bar{u}^2 + \sigma^2 \right) + (\gamma_2^2 - \gamma_1^2) \frac{2\sigma \bar{u}}{\sqrt{2\pi}} \right]. \tag{19}
\]

Equation (19) applies strictly to the unique distribution whose
mean is force 4 Beaufort. With slight modification (19) can be
applied to other distributions. If \( \sigma \) is small enough, say \( \frac{1}{2} \) Beaufort
interval, then at least 97 per cent of the distribution associated with a
mean wind of either Beaufort 3 or 5 lies on the lower or higher side,
respectively, of the critical speed (Fig. 6). Consequently, if \( \gamma_1^2 \) and
\( \gamma_2^2 \) are replaced by the appropriate single value of \( \gamma^2 \), (19) will yield
the stress with good approximation for any distribution whose mean is removed by at least $2\sigma$ from the critical speed.

With $\rho' = 1.16 \times 10^{-3} \text{ gm/cm}^3$ (corresponding to air at $25^\circ \text{C}$ and 1000 mb pressure), the equations of stress are (for $\sigma \leq \frac{1}{2}$ Beaufort interval):

$$
\tau (xy) = 0.9 \times 10^{-6} (\bar{u}^2 + \sigma^2), \quad \bar{u} \leq 3 \text{ Beaufort,} \quad (20a)
$$

$$
\tau (xy) = 2.0 \times 10^{-6} (\bar{u}^2 + 0.85 \bar{u}\sigma + \sigma^2), \quad \bar{u} = 4 \text{ Beaufort,} \quad (20b)
$$

$$
\tau (xy) = 3.0 \times 10^{-6} (\bar{u}^2 + \sigma^2), \quad \bar{u} \geq 5 \text{ Beaufort,} \quad (20c)
$$

where $\bar{u}$ and $\sigma$ are in cm/sec and $\tau$ is in dynes/cm$^2$.

![Figure 6. Assumed frequency distribution of wind speed for each integer Beaufort number of mean wind speed in the equatorial Pacific. The standard deviation of wind speed is $\frac{1}{2}$ Beaufort interval. The sea surface roughness is indicated.](image)

The value of $\sigma$ was estimated for various values of wind speed in the trade wind area, and in each case it was found to be approximately $\frac{1}{2}$ Beaufort interval (indicating an increase in the magnitude of $\sigma$ in cm/sec with increase in mean wind speed). The wind stresses for each integer Beaufort number of mean speed in the trade wind area (Table I) have been calculated on the basis of this estimate and equations (20), using the velocity conversions of the Smithsonian Tables. Stresses for $\sigma = 0$ are included for comparison.

**COMPUTATIONS**

Because of the paucity of wind observations for the periods during which the oceanic data were obtained, it is necessary to turn to cli-
matological statistics to obtain an accurate description of the distribution of wind. The data consist essentially of sixteen and eight sector wind roses from the October and November Pilot Charts (U. S. Hydrographic Office, 1946) and the September-October-November Pilot Chart (U. S. Hydrographic Office, 1942) for the Pacific. Wind roses are given for each five degree square, such that a total of 83 are included within the area concerned.

The available oceanographic observations consist principally of two sections running roughly north-south: section A, Carnegie stations 147 to 159 (Fleming, et al., 1945), occupied from October 15 to October 29, 1929, running from 138° 14' W, 27° 27' N to 159° 01' W, 9° 24' S; and section B, Carnegie stations 35 to 46, representing the period from October 26 to November 21, 1928, and Bushnell stations 307 to 299 (Sverdrup, et al., 1943), occupied from March 18 to March 25, 1939, extending from 111° 25' W, 22° 23' N to 108° 20' W, 9° 06' S. In addition, two parallel sections are included: Bushnell stations 309 to 315, extending from 124° 13' W, 29° 40' N to 149° 35' W, 20° 44' N, and Carnegie stations 131 to 137, from 126° 20' W, 33° 49' N to 145° 33' W, 24° 02' N, for April 1939 and September 1929, respectively. Fig. 13 shows the position of these stations. Values of $\Delta D$ for each station are tabulated in the reports cited.

**Wind Stress Computations.** The stresses for each direction of the wind roses were evaluated by use of Table I, according to the indicated Beaufort number. Each stress was in turn weighted according to its frequency of occurrence, and broken down into east and north components. The sum of each set of weighted components divided by

<table>
<thead>
<tr>
<th>Beaufort number of mean wind</th>
<th>Range of Velocity (mps)</th>
<th>Velocity $\bar{u}$ (cm/sec)</th>
<th>Stress, $\tau$, dynes/cm$^2$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.4-1.5</td>
<td>95</td>
<td>$\sigma = 0$</td>
</tr>
<tr>
<td>2</td>
<td>1.6-3.3</td>
<td>245</td>
<td>$\sigma = \frac{1}{2}$ Beauf. int.</td>
</tr>
<tr>
<td>3</td>
<td>3.4-5.4</td>
<td>440</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5.5-7.9</td>
<td>670</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>8.0-10.7</td>
<td>935</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10.8-13.8</td>
<td>1230</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>13.9-17.1</td>
<td>1550</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>17.2-20.7</td>
<td>1890</td>
<td></td>
</tr>
</tbody>
</table>

* The stress calculated on the basis of no standard deviation of wind speed has been included in the table (first stress column) for comparison with the values calculated for the trade wind area ($\sigma = \frac{1}{2}$ Beaufort interval).
the total frequency gave the components of mean stress. A summary of the computed stress components is presented in Tables II and III for October and November, respectively.

It is apparent from the change in sign of \( \tau_y \) that a well defined convergence of stress is present between 7½° N and 12½° N, in agreement with the convergence of mean wind streamlines analyzed by Werenskiold (1922). Zones of maximum magnitude of stress are evident at about 5° S and 17½° N, except near the continent, where the picture is obscured by the influence of the land.

The values of mean stress \( \bar{\tau}_x \) and \( \bar{\tau}_x' \) were computed for the two months using sections A and B as the arbitrary boundaries \( x_1 \) and \( x_2 \), respectively, in the averaging process. The October-November mean wind stress distribution was taken as representative of actual conditions existing at the time of the oceanographic observations (excluding Bushnell data). The computed two-month averages are shown in Fig. 7. Values of \( \tau_y \) were determined from the stresses along section A (for a 10° band of longitude). The October-November mean \( \tau_y \) curve is given in Fig. 8.

The mean longitudinal stress curves represent a total of more than 3000 observations each (U. S. Weather Bureau, 1938). The shape of these curves suggests a sinusoidal distribution of the form

\[
\tau_x = \tau_0 + a (\varphi - \varphi_0) + b \sin 2k (\varphi - \varphi_0),
\]

which would make possible an analytic solution. This will be discussed later.

---

**Figure 7.** Latitude dependence of the mean east–west wind stress components, \( \bar{\tau}_x \) and \( \bar{\tau}_x' \). The curves represent climatological average stresses for the period October 1 to November 30.
### TABLE II—SUMMARY OF COMPUTED WIND STRESSES—October

$r_x$ and $r_y$ are given in the numerator and denominator, respectively, of the tabulated fractions, the units being dynes/cm².

<table>
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<th>Lat. ($^\circ$)</th>
<th>152.5</th>
<th>147.5</th>
<th>142.5</th>
<th>137.5</th>
<th>132.5</th>
<th>127.5</th>
<th>122.5</th>
<th>117.5</th>
<th>112.5</th>
<th>107.5</th>
<th>102.5</th>
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<td>-0.02</td>
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* All values for -7.5° Latitude are September–October–November mean values (1942 Pilot chart).
TABLE III—SUMMARY OF COMPUTED WIND STRESSES—November

$\tau_x$ and $\tau_y$ are given in the numerator and denominator, respectively, of the tabulated fractions, the units being dynes/cm².

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(See Table II)
Using the values of $\tau_z$ (Fig. 7), together with its numerically determined derivatives, the evaluation of the function $f_1(\tau)$ by expression (8) was carried out for each degree of latitude between $8^\circ$ S and $22^\circ$ N (Fig. 9). Applying (9), the function $f_4(\tau)$ was evaluated for the same range (Fig. 12). The values of $\tau_z'$, together with the numerically determined second derivatives of $\tau_z'$, were used to compute the stress functions $f_2(\tau)$ and $f_3(\tau)$ by equations (6) and (7), respectively (Figs. 10, 11).

Oceanographic computations. The values of $P$ for each of the 47
stations (Table IV) were derived from equation (11) by numerical integration, using twelve standard depth intervals. The values of $d$ and $\bar{p}$ were taken as 1000 m and unity, respectively, and the constant was arbitrarily set equal to zero.

A considerable discrepancy in $P$ between the Carnegie and Bushnell stations is evident from the values of the nearly coincident runs C131 to 137 and B309 to 315 (Table IV). At a point in common with the two runs, near stations B312 and C148, it is found that the Carnegie value is $2.2 \times 10^8$ dynes/cm. in excess of that of the Bushnell.

A comparison of salinity values at great depth of the Carnegie cruise with those of other expeditions indicates that the former are consistently 0.03 parts per mille too low (Wüst, 1935; Schott, 1935; and Sverdrup, et al., 1944). This leads to an error in $P$ which is of the same sign and about the same magnitude as that found above.

**TABLE IV—List of Stations and Integrated Pressure**

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$P = \text{Integrated pressure computed directly from oceanographic data, using equation (11).}$

$P' = \text{Integrated pressure on an arbitrary scale, adjusted for a discrepancy of } 2.2 \times 10^8 \text{ dynes/cm.}$

$\text{Carnegie stations}$

$\text{Bushnell stations}$

$* \text{All values of } P \text{ and } P' \text{ are given in units of } 10^8 \text{ dynes/cm.}$
In order that the Carnegie data be compatible with the Bushnell, the quantity $2.2 \times 10^8$ dynes/cm was subtracted from each of the Carnegie values of $P$. The final values of integrated pressure, $P'$, in Table IV have been shifted to an arbitrary scale by reducing all values of $P$ by $54.0 \times 10^8$ dynes/cm. This shift has no effect upon the gradient of $P$, but it makes possible a direct comparison of computed $P$ with that found by the analytic solution to be discussed later.

A semiobjective procedure was followed in evaluating the average gradient of $P$ in the $x$ direction. Since the values of $P$ for section A form a remarkably smooth curve (Fig. 10), the differences, $\Delta P$, be-

![Figure 9](image-url)
Figure 10. Plot of theoretical and observed values of $\partial P/\partial y$ as a function of latitude. Observed values have been evaluated from the Carnegie data along section A. Values of $P$ for each station of section A are included.

Between the sections were computed at latitudes corresponding to those of the stations of the eastern section. Interpolation was necessary at section A only. Values of $\Delta x$ were found in a similar manner, and several values of $\Delta P/\Delta x$ derived (Fig. 9).

Section A was used to obtain discrete values of the north–south gradient of $P$ between stations. In determining the approximations of $\partial P/\partial y$, account was taken of the fact that this section is not truly north–south. The gradient is given by the following approximation:
Figure 11. Latitude dependence of the longitudinal mass transport, $M_x$, computed by two independent methods. $M_x$ represents the net eastward transport of water in tons per second through a column of 1000 meters depth and 1 meter width, at the Carnegie path (section A).
Figure 12. Latitude dependence of the latitudinal mass transport, $M_Y$, computed by two independent methods. $M_Y$ represents the mean net northward transport of water in tons per second through a column of 1000 meters depth and 1 meter width, between the two sections A and B.
where $\Delta P = \Delta P \left( x_1 y \right)$ is the difference of $P$ between adjacent stations, $\Delta y$ is the corresponding change of $y$, and $\beta$ is the bearing of the path. The gradient $\partial P / \partial x$ was approximated by $\Delta P / \Delta x$ in the determination of the final north–south gradient of $P$ (Fig. 10).

The values of $\Delta P / \Delta x$ and $\partial P / \partial y$ for latitudes, $|\phi|$, greater than $1^\circ$ were employed in computing $M_x$ and $M_y$ by means of (12). Values within one degree of the equator were used in conjunction with adjacent values of the gradients to determine the second derivatives of $P$ at the equator. This afforded means of computing the transport at the equator by (13). The resulting values are plotted in Figs. 11 and 12.

**ANALYTIC SOLUTION**

The essential features of the horizontal distribution of transport and of mass in the equatorial Pacific can be recognized most readily by an examination of the following simple wind stress distribution:

$$
\tau_x(y) = \tau_0 + b \sin 2k (\phi - \phi_0),
$$

and

$$
\tau_y(y) = -c \sin k (\phi - \phi_0'),
$$

where $\tau_0, b, c, k, \phi_0, and \phi_0'$ are constants. Equation (23a) is a special case of (21).

The boundary is approximated [for the range $-15^\circ \leq \phi \leq 35^\circ$] by the analytic function:

$$
x_0(y) = L - my + n \sin 2K (\phi - \phi_0''),
$$

or

$$
\tan \theta = \frac{dx_0}{dy} = -m + \frac{2Kn}{R} \cos 2K (\phi - \phi_0''),
$$

where $L, m, n, K, and \phi_0''$ are constants.

Equations (1) and (5), together with the expressions (23) and (24), determine an analytic function $P = F(xy)$. The complete solution can be shown to be:

$$
F(xy) = \tau_0 x + \left[ x - x_0(y) \right] \left\{ b \sin 2k(\phi - \phi_0) - 2kb \cos 2k(\phi - \phi_0) \tan \phi \right\} \\
+ \frac{cR}{k} \cos k(\phi - \phi_0') + \frac{bRm}{2k} \cos 2k(\phi - \phi_0)
$$
\[
\frac{bnK}{(k^2 - K^2)} \left[ K \cos 2k(\varphi - \varphi_0) \cos 2K(\varphi - \varphi_0'') + K \sin 2k(\varphi - \varphi_0) \sin 2K(\varphi - \varphi_0'') \right],
\]

provided that \( k^2 \neq K^2 \). The constant of integration is arbitrarily set equal to zero.

It can be further verified that the mass transport components, satisfying equations (4), (5), (23), and (24), are given by the expressions:

\[
M_x = -\frac{2k^2b}{\omega R} \left[ x - x_0(y) \right] \sin [2k(\varphi - \varphi_0) - \alpha] \cos \varphi \cos \alpha
+ \frac{k_b \cos 2k(\varphi - \varphi_0)}{\omega \cos \varphi} \left[ m - \frac{2Kn}{R} \cos 2K(\varphi - \varphi_0'') \right],
\]

and

\[
M_y = -\frac{k_b \cos 2k(\varphi - \varphi_0)}{\omega \cos \varphi}.
\]

In (25) and (26) the term \( x \) replaces the term \( x_1 \) in the original equations. The angle \( \alpha \) is a small fraction of \( \varphi \), according to the relation

\[
\tan \alpha = \frac{\tan \varphi}{2k}.
\]

For \((x - x_0) \geq R\), the last three terms of (25) and the last term of (26a) are unimportant.

Since the field of mass is assumed stationary, implying that the mass transport is nondivergent, there exists a stream function, \( \psi = \psi(xy) \), for which

\[
\frac{\partial \psi}{\partial x} = -M_y, \quad \frac{\partial \psi}{\partial y} = M_x.
\]

In view of (26), the solution for \( \psi \) is

\[
\psi = \frac{k_b}{\omega} \left[ x - x_0(y) \right] \frac{\cos 2k(\varphi - \varphi_0)}{\cos \varphi}.
\]

It can be shown that (28) holds for any boundary.

The transport of water between the surface and level \( d \) is parallel to the isolines of \( \psi \), or streamlines, and the mass flowing per unit time between adjacent streamlines is equal to the corresponding difference in \( \psi \).

The constants were taken as follows:
Figure 13. The horizontal distribution of integrated pressure, $F(xy)$, based on a sinusoidal distribution of longitudinal wind stress and an hypothetical boundary (see p. 92). The values of adjusted integrated pressure, $P'$, computed from oceanographic data, are plotted for comparison.
Figure 14. Streamlines of flow, representing the field of mass transport, based on a sinusoidal stress distribution and the actual boundary. Values of the stream function, \( \psi \), are indicated. Differences of \( \psi \) between streamlines represent the net mass in millions of tons flowing per second between those streamlines (from the surface to a depth of 1000 meters). The boundary represents a streamline for which \( \psi = 0 \).
\( \tau_0 = -0.3 \text{ dyne/cm}^2, \ m = 0.85, \ R = 6.37 \times 10^8 \text{ cm}, \)

\( b = 0.3 \text{ dyne/cm}^2, \ n = 9 \times 10^7 \text{ cm}, \ \varphi_0 = 1^\circ, \)

\( c = 0.4 \text{ dyne/cm}^2, \ k = 180/26 = 6.923, \ \varphi_0' = \varphi_0 + 90^\circ/2k = 7.5^\circ, \)

\( L = 80^\circ \text{ W longitude}, \ K = 180/40 = 4.500, \ \varphi_0'' = -2^\circ. \)

Values of \( F(xy) \) from (25) were determined for the region from 160° W longitude to the analytic boundary (Fig. 13). These were compared with corresponding values of \( P \), computed from oceanic data, enabling the reduction of \( P \) to a scale in common with \( F(xy) \). The adjusted values, \( P' = P - 54.0 \times 10^8 \text{ dynes/cm} \) (Table IV), have been plotted in the horizontal distribution chart (Fig. 13). A coefficient of correlation of 0.73 between the analytic solution, \( F \), and the values of \( P' \) is obtained for the stations cited.

The quantity \( F(xy) \) or \( P \) has a physically clear meaning in the case of the simple two layer system, where a light homogeneous layer of thickness \( h \) overlies a heavier homogeneous layer at rest. In this case, it can be demonstrated (Defant, 1936) that \( P \) is a quadratic in \( h \). For such a system then, \( h \) would increase with distance from the coast.

Fig. 14 is a plot of the stream function based on the true boundary. It can be seen that the total mass transport of the counter current across the 140th meridian, from the difference of \( \psi \), is about 35 million tons per second. This is somewhat higher than found for both the oceanographic and actual stress data. The boundary represents a zero valued streamline, but not an isoline of \( F(xy) \).

**DISCUSSION**

Figs. 9 and 10 are similar to those discussed by Sverdrup (1947). Despite the fact that the oceanic data are meager, the curves show a remarkable agreement with the principal characteristics of those observations.

The comparison of the mass transports, computed by the two independent methods, is even more striking (Figs. 11, 12). The total mass transport of the equatorial counter current, between the surface and 1000 meters computed by numerical integration of \( f_3 (\tau) \), is 26.7 million metric tons per second, across the Carnegie section. This compares well with the estimate of transport given by Sverdrup, *et al.* (1942), which is 25 million m³ per second between the surface and 700 meters, for the same section.

The limits of the counter current are at 2° N and 12° N latitude (Fig. 11), and are indicative of the mean width of the current for the 1000 meter column. However, because of the widening of the current
with depth, indicated by the Carnegie profile (Sverdrup, et al., 1942),
the surface current should be of less width. In support of this, the
October and November Pilot Charts (U. S. Hydrographic Office, 1946)
show surface currents to the east in a band from about 5° N to 12° N.
From the plots of $P$ (Figs. 10, 13) it is evident that the principal
troughs and ridges are accounted for by the wind stress functions,
except in the region of the equator, where the theory fails to describe
the conspicuous rise of $P$ to the north across the equator. It is
possible that this anomalous pressure gradient could be maintained by
thermodynamic processes in the vicinity of the equator. This is
suggested by the fact that the observed maximum in $P$, just north of
the equator, is almost coincident with the minimum in net evaporation
given by Wüst (1936). It is clear that such a gradient, in excess
of that which can be maintained by the stress of the wind at the
equator, can be balanced only by lateral friction forces if unaccelerated
motion is to prevail. For this reason, the picture of flow in the vicinity
of the equator is not clear.
An alternate possibility is that the discrepancy mentioned above is
not a real one. The mean atmospheric pressure distribution for
October and November is asymmetrical with respect to the equator.
This could bring about a discontinuity in longitudinal stress, and in
turn a sharp rise of the function $P$ across the equator. This is
apparent if one considers the following approximation obtained from
(1a) and (7):
\[
P = (x_1 - x_0) \left( \tilde{\tau}_x' - R \tan \varphi \frac{\partial \tilde{\tau}_x'}{\partial y} \right) + \text{constant}. \quad (29)
\]
This expression is meaningless at $\varphi = 0$, if $\tilde{\tau}_x'$ is discontinuous, but at a
very small distance from the equator the expression reduces to
$(x_1 - x_0) \tilde{\tau}_x'$. Accordingly, a sharp rise of but 0.2 dyne/cm$^2$ in $\tilde{\tau}_x'$
across the equator could lead to a corresponding rise of $P$ in the
amount of $1.0 \times 10^8$ dyne/cm along the Carnegie section.

SUMMARY AND CONCLUSION

By ignoring lateral friction and thermodynamic processes, it has
been verified that, in the eastern equatorial Pacific, the predominant
features of the distribution of mass and of mass transport of water can
be accounted for semiquantitatively in terms of the wind stress alone.
The longitudinal mass transport increases nearly linearly with
distance from the continent, while the north–south transport is
practically independent of longitude and is a maximum very near the
boundaries of the counter current.
The principal discrepancy of theory and observation exists in the proximity of the equator, where it is possible that thermodynamic processes and lateral friction are important. This discrepancy may not be real but may be brought about because of insufficient details of wind stress near the equator.

No attempt has been made to describe the vertical distribution of mass; such an analysis is to be presented in a forthcoming paper.

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