The *Journal of Marine Research* is an online peer-reviewed journal that publishes original research on a broad array of topics in physical, biological, and chemical oceanography. In publication since 1937, it is one of the oldest journals in American marine science and occupies a unique niche within the ocean sciences, with a rich tradition and distinguished history as part of the Sears Foundation for Marine Research at Yale University.

Past and current issues are available at [journalofmarineresearch.org](http://journalofmarineresearch.org).

Yale University provides access to these materials for educational and research purposes only. Copyright or other proprietary rights to content contained in this document may be held by individuals or entities other than, or in addition to, Yale University. You are solely responsible for determining the ownership of the copyright, and for obtaining permission for your intended use. Yale University makes no warranty that your distribution, reproduction, or other use of these materials will not infringe the rights of third parties.

This work is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. To view a copy of this license, visit [http://creativecommons.org/licenses/by-nc-sa/4.0/](http://creativecommons.org/licenses/by-nc-sa/4.0/) or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.
TABLE FOR DENSITY OF SEA WATER AT 0° C. AND ATMOSPHERIC PRESSURE FOR VALUES OF CHLORINITY BETWEEN 18.00 AND 20.99 ACCORDING TO KNUDSEN’S FORMULA

BY

R. B. MONTGOMERY AND R. B. SYKES, JR.

Woods Hole Oceanographic Institution

It is customary to express the density of sea water at atmospheric pressure,

\[ 1 + \sigma_t \cdot 10^{-3}, \]

in terms of \( \sigma_t \) to two decimal places. This is usually computed from chlorinity and temperature by means of the Hydrographical Tables (Knudsen, 1901), which give it as the difference of two terms:

\[ \sigma_t = \sigma_0 - D. \]

\( \sigma_0 \) is given as a function of chlorinity alone, the density of sea water at 0° C. and atmospheric pressure being

\[ 1 + \sigma_0 \cdot 10^{-3}, \]

and \( D \) is tabulated as a function of \( \sigma_0 \) and temperature. While \( D \) is tabulated to three decimal places, \( \sigma_0 \) is given to only two decimal places in the Tables, so their use is a departure from the common rule that calculations be executed to at least one more significant figure than the result.

Of course there is necessarily an uncertainty in the second decimal place of \( \sigma_t \) due to the uncertainty in chlorinity and temperature as commonly measured. This probably produces an error in \( \sigma_t \) which very often amounts to 0.01 and occasionally to even 0.03. Nevertheless this is no reason to introduce a further uncertainty due merely to calculation. This is especially true in deep water, where the second decimal place may assume great importance so that its highest accuracy in keeping with convenience is desirable. If \( \sigma_0 \) and \( D \) are known exactly to the second decimal place only, the resulting value of \( \sigma_t \) may be in error by as much as 0.010 and the chances are one in four that it is not correct in the second decimal place. Furthermore chlorinity is sometimes determined to some significance in the third

\(^{1}\) Contribution No. 275.
decimal place (of %\text{O} ), and this cannot be adequately converted to \( \sigma_0 \) when the latter is tabulated to two decimal places only.

In order to eliminate largely this error of calculation we have formed the accompanying table giving \( \sigma_0 \) to three decimal places for the range of chlorinity from 18.00 to 20.99\%, which fully covers the waters found in the open ocean. It was computed from Knudsen's formula\(^2\) relating \( \sigma_0 \) to chlorinity in parts per thousand,

\[
\sigma_0 = -0.069 + 1.4708 \ Cl - 0.001570 \ Cl^2 + 0.0000398 \ Cl^3.
\]

In regard to the accuracy of the formula it is first to be noted that, even if the coefficients are exact in the last figure given, a maximum error in \( \sigma_0 \) of 0.0021 at chlorinity 20\%\text{O} might result because the subsequent figures are unknown. The coefficients were derived by Knudsen (1902, p. 157) from 22 selected samples for which the chlorinity and density were carefully determined. The average magnitude of the deviations of the \( \sigma_0 \) values from the formula is 0.0047; this was assigned chiefly to errors in the chlorinity determinations. The chlorinity values used by Knudsen, according to the old definition of chlorinity (Sørensen, 1902, p. 137) but using 1938 atomic weights, need to be increased by a factor 1.00048, the standard water of supposed chlorinity 19.380%\text{O} having an actual value 0.0094%\text{O} greater as given by Jacobsen and Knudsen (1940, p. 22). In order to obviate this discrepancy they have redefined chlorinity as 0.3285234 times the mass of silver necessary to precipitate the halogens in unit mass of sea water, and the formula for \( \sigma_0 \) is appropriate when chlorinity is thus defined.

A check on the formula at one point was obtained by the precise analysis of the Urnormal 1937 water. This sample of natural sea water was found to have a chlorinity of 19.381%\text{O} and \( \sigma_0 = 28.137 \), the latter agreeing exactly with the formula (Jacobsen and Knudsen, 1940, p. 38). The conclusion indicated is that within the range of the accompanying table the third decimal place of \( \sigma_0 \) as given by the formula is doubtful but at the same time definitely significant even in absolute value. Of course it is assumed implicitly in the formula that variations in concentration of other dissolved substances relative to chlorine are too small to make the third decimal place of \( \sigma_0 \) insignificant. While this is justified for the samples Knudsen worked with, no sufficiently extended study has been made to ensure that the simple formula holds for all ocean waters to this degree of accuracy.

\(^2\) Each value before rounding off to three decimal places was found to agree with the Hydrographical Tables with the exception of chlorinity 18.94\%, for which he gives \( \sigma_0 = 27.49 \), the formula giving 27.4952.
In regard to the values of $D$ in the Hydrographical Tables, Knudsen (1902, p. 172) points out that linear interpolation may lead to an error as great as 0.002, but that the tabulated values are fully exact in the third decimal place. It is however doubtful that sufficiently diversified samples of sea water were investigated to be sure of this statement.

REFERENCES

JACOBSEN, J. P., and MARTIN KNUDSEN.
1940. Urnormal 1937 or Primary Standard Sea-Water 1937. Association d'Océanographie Physique, Union Géodésique et Géophysique Internationale, Publication Scientifique, 7 (38 pp.).

KNUDSEN, MARTIN.

SØRENSEN, S. P. L.
<table>
<thead>
<tr>
<th>Cl,°/oo</th>
<th>.00</th>
<th>.01</th>
<th>.02</th>
<th>.03</th>
<th>.04</th>
<th>.05</th>
<th>.06</th>
<th>.07</th>
<th>.08</th>
<th>.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.0</td>
<td>26.129</td>
<td>143</td>
<td>158</td>
<td>172</td>
<td>187</td>
<td>201</td>
<td>216</td>
<td>231</td>
<td>245</td>
<td>260</td>
</tr>
<tr>
<td>18.1</td>
<td>26.274</td>
<td>289</td>
<td>303</td>
<td>318</td>
<td>332</td>
<td>347</td>
<td>361</td>
<td>376</td>
<td>390</td>
<td>405</td>
</tr>
<tr>
<td>18.2</td>
<td>26.419</td>
<td>434</td>
<td>449</td>
<td>463</td>
<td>478</td>
<td>492</td>
<td>507</td>
<td>521</td>
<td>536</td>
<td>550</td>
</tr>
<tr>
<td>18.3</td>
<td>26.565</td>
<td>579</td>
<td>594</td>
<td>608</td>
<td>623</td>
<td>637</td>
<td>652</td>
<td>667</td>
<td>681</td>
<td>696</td>
</tr>
<tr>
<td>18.4</td>
<td>26.710</td>
<td>725</td>
<td>739</td>
<td>754</td>
<td>768</td>
<td>783</td>
<td>797</td>
<td>812</td>
<td>826</td>
<td>841</td>
</tr>
<tr>
<td>18.5</td>
<td>26.855</td>
<td>870</td>
<td>885</td>
<td>899</td>
<td>914</td>
<td>928</td>
<td>943</td>
<td>957</td>
<td>972</td>
<td>986</td>
</tr>
<tr>
<td>18.6</td>
<td>27.001</td>
<td>915</td>
<td>930</td>
<td>944</td>
<td>959</td>
<td>974</td>
<td>988</td>
<td>1003</td>
<td>1017</td>
<td>1032</td>
</tr>
<tr>
<td>18.7</td>
<td>27.146</td>
<td>1061</td>
<td>1075</td>
<td>1090</td>
<td>1104</td>
<td>1119</td>
<td>1133</td>
<td>1148</td>
<td>1162</td>
<td>1177</td>
</tr>
<tr>
<td>18.8</td>
<td>27.292</td>
<td>1206</td>
<td>1221</td>
<td>1235</td>
<td>1250</td>
<td>1264</td>
<td>1279</td>
<td>1293</td>
<td>1308</td>
<td>1322</td>
</tr>
<tr>
<td>18.9</td>
<td>27.437</td>
<td>1352</td>
<td>1367</td>
<td>1381</td>
<td>1396</td>
<td>1410</td>
<td>1425</td>
<td>1439</td>
<td>1454</td>
<td>1469</td>
</tr>
<tr>
<td>19.0</td>
<td>27.582</td>
<td>1501</td>
<td>1516</td>
<td>1530</td>
<td>1545</td>
<td>1559</td>
<td>1574</td>
<td>1588</td>
<td>1603</td>
<td>1618</td>
</tr>
<tr>
<td>19.1</td>
<td>27.728</td>
<td>1652</td>
<td>1667</td>
<td>1681</td>
<td>1696</td>
<td>1710</td>
<td>1725</td>
<td>1739</td>
<td>1754</td>
<td>1769</td>
</tr>
<tr>
<td>19.2</td>
<td>27.873</td>
<td>1805</td>
<td>1820</td>
<td>1834</td>
<td>1849</td>
<td>1863</td>
<td>1878</td>
<td>1892</td>
<td>1907</td>
<td>1922</td>
</tr>
<tr>
<td>19.4</td>
<td>28.164</td>
<td>2112</td>
<td>2127</td>
<td>2141</td>
<td>2156</td>
<td>2170</td>
<td>2185</td>
<td>2200</td>
<td>2214</td>
<td>2229</td>
</tr>
<tr>
<td>19.5</td>
<td>28.310</td>
<td>2267</td>
<td>2282</td>
<td>2297</td>
<td>2311</td>
<td>2326</td>
<td>2340</td>
<td>2355</td>
<td>2369</td>
<td>2384</td>
</tr>
<tr>
<td>19.6</td>
<td>28.455</td>
<td>2423</td>
<td>2438</td>
<td>2452</td>
<td>2467</td>
<td>2482</td>
<td>2497</td>
<td>2511</td>
<td>2526</td>
<td>2541</td>
</tr>
<tr>
<td>19.7</td>
<td>28.601</td>
<td>2580</td>
<td>2595</td>
<td>2609</td>
<td>2624</td>
<td>2639</td>
<td>2654</td>
<td>2669</td>
<td>2684</td>
<td>2699</td>
</tr>
<tr>
<td>19.8</td>
<td>28.746</td>
<td>2737</td>
<td>2752</td>
<td>2767</td>
<td>2782</td>
<td>2797</td>
<td>2812</td>
<td>2827</td>
<td>2842</td>
<td>2857</td>
</tr>
<tr>
<td>19.9</td>
<td>28.892</td>
<td>2894</td>
<td>2909</td>
<td>2924</td>
<td>2939</td>
<td>2954</td>
<td>2969</td>
<td>2984</td>
<td>3000</td>
<td>3015</td>
</tr>
<tr>
<td>20.0</td>
<td>29.037</td>
<td>3052</td>
<td>3067</td>
<td>3081</td>
<td>3096</td>
<td>3111</td>
<td>3126</td>
<td>3141</td>
<td>3156</td>
<td>3171</td>
</tr>
<tr>
<td>20.1</td>
<td>29.183</td>
<td>3212</td>
<td>3227</td>
<td>3241</td>
<td>3256</td>
<td>3271</td>
<td>3286</td>
<td>3301</td>
<td>3316</td>
<td>3331</td>
</tr>
<tr>
<td>20.2</td>
<td>29.329</td>
<td>3372</td>
<td>3387</td>
<td>3402</td>
<td>3417</td>
<td>3432</td>
<td>3447</td>
<td>3462</td>
<td>3477</td>
<td>3492</td>
</tr>
<tr>
<td>20.3</td>
<td>29.474</td>
<td>3532</td>
<td>3547</td>
<td>3562</td>
<td>3577</td>
<td>3593</td>
<td>3608</td>
<td>3623</td>
<td>3638</td>
<td>3653</td>
</tr>
<tr>
<td>20.4</td>
<td>29.620</td>
<td>3694</td>
<td>3709</td>
<td>3724</td>
<td>3739</td>
<td>3754</td>
<td>3769</td>
<td>3784</td>
<td>3799</td>
<td>3814</td>
</tr>
<tr>
<td>20.5</td>
<td>29.765</td>
<td>3857</td>
<td>3872</td>
<td>3887</td>
<td>3902</td>
<td>3917</td>
<td>3932</td>
<td>3947</td>
<td>3962</td>
<td>3977</td>
</tr>
<tr>
<td>20.6</td>
<td>29.911</td>
<td>4020</td>
<td>4035</td>
<td>4050</td>
<td>4065</td>
<td>4080</td>
<td>4095</td>
<td>4110</td>
<td>4125</td>
<td>4140</td>
</tr>
<tr>
<td>20.7</td>
<td>30.057</td>
<td>4184</td>
<td>4200</td>
<td>4215</td>
<td>4230</td>
<td>4245</td>
<td>4260</td>
<td>4275</td>
<td>4290</td>
<td>4305</td>
</tr>
<tr>
<td>20.8</td>
<td>30.203</td>
<td>4348</td>
<td>4363</td>
<td>4378</td>
<td>4393</td>
<td>4408</td>
<td>4423</td>
<td>4438</td>
<td>4453</td>
<td>4468</td>
</tr>
<tr>
<td>20.9</td>
<td>30.348</td>
<td>4513</td>
<td>4528</td>
<td>4543</td>
<td>4558</td>
<td>4573</td>
<td>4588</td>
<td>4603</td>
<td>4618</td>
<td>4633</td>
</tr>
</tbody>
</table>