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ON THE VALIDITY OF THE DYNAMIC TOPOGRAPHIC METHOD FOR THE DETERMINATION OF OCEAN CURRENT TRAJECTORIES

BY

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In a previously published statement (Parr 1937, b) the writer has listed the main reasons why, in the present state of our knowledge of the actual as distinct from the ideal dynamics of ocean currents, he feels inclined to take greater confidence in the evidence of the actual paths of flow obtained from a study of the identifying properties of the water masses, than in the conclusions reached by present efforts to deduce the trajectories of motion from hydrodynamic calculations and considerations. In this article a further elaboration and elucidation of these objections against the unrestricted use of dynamic methods for the determination of ocean current trajectories will be attempted, while suggestion for obtaining a presumably closer approach to an accurate presentation of flow through isopycnic analysis by means of identifying properties will be presented in the next article in this issue.

It should be clearly understood, however, that the objections against the dynamic methods here advanced are not directed against the theoretical validity of Bjerkness' circulation theorem under ideal abstract circumstances, but against the practical application of this theorem to actual conditions in nature, on the working hypothesis that the direction of flow will always be at right angles to the dynamic gradient (or parallel with the dynamic contour lines) with general disregard from the question of whether these actual conditions adequately fulfill the theoretical requirements for a valid application of the method.

With reference to the dangers inherent in the unrestricted use of dynamic topographies on the basis of the practical working hypothesis above mentioned, the writer has already summarized his objections in the statement "that the dynamic methods of analysis deal only with states of motion and not with the trajectories or paths of transportation, and, since the error in the calculations for states of motion is cumulative with reference to trajectories, an accuracy which is adequate for approximate determination of states of motion may be utterly inadequate for analysis of actual paths of transportation" (Parr, 1937 b). In the following we shall consider the reasons why it seems probable that it will prove the exception rather than the rule under natural conditions to find the ideal requirements for the
dynamic method sufficiently fulfilled to make the accuracy of this method adequate also for the study of current trajectories.

The first and most general difficulty arises from the fact that dynamic topographies can only deal with components of motion in the two horizontal dimensions, but are incapable of including the third dimension in their presentation of the probable current picture. In this simple fact, we find an objection of universal character, since all liquid flow is basically three-dimensional and can never find full expression in a two-dimensional presentation except in the form of a rough approximation where the vertical components are negligible. If we nevertheless arbitrarily select to assume that the contour lines are indeed identical with the actual lines of flow, we not only disregard the probable existence of vertical components of motion, but actually by our assumption rule out the possibility of any vertical movement whatsoever (except by the process of diffusion), since the dynamic contour lines must always form closed curves* therefore presenting for each isobaric surface a completely self-contained and self-compensating circulation system, not permitting of any invasion from or evasion to other isobaric surfaces. True convergence or true divergence can therefore never become apparent in the dynamic topographies alone, and the cases in which the dynamic interpretation becomes most obviously unsatisfactory are therefore the cases in which these phenomena are definitely known to occur.

While the successful prediction of the movement of icebergs accomplished by the U. S. Ice Patrol from a study of dynamic topographies is one of the greatest practical achievement of applied oceanography, of immeasurable value to human lives, it must be remembered that unless the liquid flow itself is strictly confined to two dimensions only, the movements of a rigid body floating in, or on, a complicated liquid current system will only express a mechanical integration of the horizontal components of the liquid flow upon which the object is carried, without any direct relation to the actual liquid trajectories. Comparison might be made with the movements of an object transported on a conveyor system of rollers rotating in the same direction. The movement of the object would express the mechanical integration of direction and velocities of the movement of the upper roller surfaces reduced to two dimensions, but could not be taken as an indication of the nature of the roller motion itself, unless its mechanism was known to us from entirely different sources of information.

Both because the conditions at the meeting of the Labrador Current and the so-called Gulf Stream in the vicinity of the Newfoundland banks presents us with perhaps the most unquestionable case of a true convergence, undoubtedly at least in considerable part involving movements in three

* Apart from the definitely not ideal cases to be dealt with later, in which the dynamic contour lines have their end point or their point of origin at the shore.
dimensions, and also for the very reason of the successful application of the
dynamic topographic method to the *approximate* prediction of the mecha-
nical integration, in the movements of the icebergs, of the two-dimensional
components of this three-dimensional flow; the hydrographic observations
recorded by the U. S. Ice Patrol may provide us with a particularly suitable
example for a study of the relationship between actual liquid trajectories
and the dynamic contour lines, when movement occurs in more than two
dimensions.

![Diagram of ice patrol stations]

**Figure 36.** Surface isotherms for 10°C. and 6°C. (---·---·) and intermediate isotherm
for 4°C. (- · - · - · ) superimposed upon dynamic topography of free surface relative to 750
decibars, according to U. S. Ice Patrol observations for June 25–29, 1926 (Smith, 1927).

The report on the international ice observations for the season of 1926
(Smith 1927) gives the data on which the accompanying Fig. 36 is based.
In this figure, representing conditions during June 25–29, 1926, the dynamic
contours are drawn in light solid lines according to the interpretation of
Smith (1927, Fig. 57, p. 115). Superimposed upon this dynamic topography
in heavy dot and dash are the surface isotherms for 10°C. and 6°C., and in
heavy dotted lines the southern limits of occurrence of water colder than
4°C. as an *intermediate* layer below the surface. In this picture we find some
agreement in broad generalities between the distribution of surface tem-
peratures and the trajectories indicated by the contour lines, but a glaring
discrepancy in the details, particularly when we notice how the contour
lines coming in from the north with cold surface waters of northern origin having temperatures of less than 6° C. and salinities of less than 33°/oo, depart again to the north in what would purport to be a continuous trajectory carrying warm surface waters of unquestionably southern derivation with temperatures of more than 10° C. and salinities of nearly 34°/oo. On the assumption that contour lines and true trajectories should be identical, this transfer of surface waters should be utterly impossible. But it is evident that surface waters of southern origin must have traversed the contour lines towards the north, that is, they must have descended a dynamic gradient calculated with reference to a fixed datum level, to a considerable extent. If it should be felt that the magnitude of the discrepancy in terms of miles, within the small area illustrated, it not so exceedingly great, it should be kept in mind, first, that the discrepancy must actually be greater than indicated by the isotherms (see below) and, secondly, that even if we assume that actual trajectories, after this region of convergence has been passed, will approximately follow the contour lines in their further course across the North Atlantic, an error of only a few miles from one trajectory to another within this small area of convergence may amount to an error of hundreds of miles in the eastern part of the ocean.

The discrepancy patently arises from the fact that we here have a true convergence with the actual trajectories of the incoming current from the north submerging under the surface trajectories of the supernatant current from the east and south. Since the dynamic contour lines do, and as above mentioned always must, enter and leave the picture in equal numbers, we here have an excellent illustration of the failure of the contour lines to indicate true trajectories in the presence of significant vertical components.

That we are actually dealing with a true submergence may be seen by the extent of the southward distribution of the intermediate layer colder than 4° C. far beyond the northward distribution of the surface waters of southern origin warmer than 10° C. In making this comparison, one must keep in mind that the illustration undoubtedly greatly minimizes the extent of the overlap between southern and northern waters, due to the fact that both limiting isotherms have been pushed back towards their region of origin in relation to the actual extent of the mutual invasions which gave rise to their distribution; the limiting isotherm for surface waters warmer than 10° C. by chilling from the atmosphere and from the waters below, that of the intermediate water colder than 4° C. by warming both from above and from below.

The actual existence of this convergence and submergence of northern waters is, of course, already well known and recognized by all investigators of North Atlantic circulation, and it can be even better illustrated than in Fig. 36 by the observations made by the Ice Patrol somewhat earlier in the same season (May 14–20, 1926, Smith, 1927). These observations are plotted in Fig. 37, in which the surface isotherm of 4° C., in heavy solid line,
and the limits of southern subsurface penetration of waters colder than 4°C. at intermediate levels, in heavy dotted line, are superimposed upon the dynamic topography of the surface. In this case the descent of water colder than 4°C. from the free surface has occurred within the region of observation, and the continuity of the descent can be followed. If we make a composite profile through the region of greatest southward penetration of this intermediate submerged tongue of northern water, by projecting the sta-

Figure 37. Surface isotherm for 4°C. (-----) compared with isotherm for 4°C. at intermediate depths (· · · · · ·) superimposed upon dynamic topography of free surface relative to 750 decibars according to U. S. Ice Patrol observations for May 14–20, 1926 (Smith, 1927). Insert: arrangement of composite profile shown in fig. 38.

tions within this tongue to a straight line as shown in the insert in Fig. 37, we obtain the picture given in Fig. 38 (with the individual station curves for temperature shown at the right). From this dichotomous picture it is rather clearly indicated that both types of submergence to be expected in the neighborhood of melting ice must have occurred in the vicinity of Newfoundland Bank, namely an almost vertical convection of bottom water, and an oblique submergence of intermediate water, both of northern origin. This, one will notice, corresponds very closely and interestingly to conditions in the Antarctic (see Wüst, 1928; Sverdrup, 1933), although on a much smaller scale.

From these pictures it is evident that there must be an oblique move-
ment from the surface downwards and southwards, and this invasion of heavier water from the north at intermediate levels must cause a decrease in the dynamic elevation of the surface calculated with reference to a fixed datum level, which will be at variance with the actual northward extension of the surface trajectories.

When the Ice Patrol reports repeatedly mention cases in which the icebergs actually seemed to move almost at right angles to the dynamic contour lines, this is undoubtedly often, at least in part, due to the fact that the more deeply submerged portions of the berg are still being forced southward by the submerging Arctic waters. In one instance when such transverse movements are reported (Stations 1109–1111, International Ice Observations, Season of 1930) two of the three stations in the series concerned actually show a strong density inversion associated with the intermediate submergence of northern waters, each inversion confirmed by two observations in the same vertical. This is a condition which can only exist as a transitory state in the presence of high velocities of transverse movement in the intermediate cold layer.

On the whole, conditions at the southern end of the Newfoundland Bank would thus seem to furnish an excellent example of the inadequacy of dynamic topographies for the interpretation of current trajectories in regions of true convergence.
A further difficulty which may become important in special cases, but which is of less general significance, is caused by the influence of centrifugal forces whenever there is a change in the radius of curvature of the current trajectories. As previously stated (Parr 1937b) strictly ideal contour line flow can even under otherwise ideal circumstances only occur in a straight parallel, or in a concentric circular system. Here again we may find an example in the U. S. Ice Patrol observations. Fig. 39, comparing dynamic topography with recorded movements of icebergs, is taken from Smith (1931, Fig. 106, p. 165) and shows a centrifugal tendency of the bergs in relation to the contour lines with a rapidly changing radius of curvature.

Since the effect of the transverse movement of submerged water also enters into the situation (see p. 124), it is far from being a clear case, but centrifugal forces must obviously have contributed to some extent. If, in other instances similar abrupt changes of direction occur without true convergence, the moving water masses must themselves be under the influence of similar centrifugal forces.

A third objection to the expectation that actual conditions in nature will normally, and not only exceptionally, fulfill the requirements for an ideal flow parallel with the dynamic contour lines, is to be found in the fact that the assumption of approximately ideal flow does not take into consideration the loss or gain of dynamic height suffered internally in the direction of motion due to changes in specific gravity caused within the moving mass of water itself by heating or cooling, etc. This objection has already previously (Parr 1936a) been elaborated in greater detail with reference to the changes caused by climatic influences operating from the surface, and a possible
method for obtaining approximate corrections for this factor has been suggested. One might add the statement at this point that the effects of changes in density in the moving mass caused by diffusion from adjacent bodies of water should be similar to those arising from external chilling or heating, and perhaps of even greater general significance.

If, as an abstracting sample we consider that a jet-stream is actually, for whatever dynamic reason, being ejected from a warm body of water into a colder body of similar chemistry (or into a colder region), it is obvious that this jet would suffer an internal contraction and a gain in specific density,

![Diagram of contour lines and trajectories in a jet-stream from a warm to a cold region on the northern hemisphere.](image)

Figure 40. Comparison of contour lines and trajectories in a jet-stream from a warm to a cold region on the northern hemisphere.

with consequent loss of dynamic height, and we would have a situation such as that illustrated in Fig. 40, in which the calculated dynamic contour lines would run at right angles to the actual lines of flow in the middle of the current, and in which the error with regard to direction of motion developing from the assumption of contour-line trajectories in extreme cases might approach 180° on the right side of the current (left side on the southern hemisphere). It seems increasingly evident that the jet-stream type of current (Rossby, 1936) may play a very important role in the general oceanic circulation, and such discrepancies between dynamic contour lines and actual current trajectories as those indicated in this example may therefore have a far more common occurrence than heretofore realized. An apparently excellent case in point, taken from actual observations, may be seen when the dynamic contour lines drawn by Jacobsen (1929, Fig. 41, in
this article) for the region of approach to the British waters is compared with the current map for the North Atlantic prepared by Meyer (1928) from empirical data, and of which a fragment is here reproduced (Fig. 42), with selected contour lines according to Jacobsen entered to facilitate the comparison, which otherwise hardly requires any further elaboration.* Undoubtedly still better illustrations of this particular point could be found in the nearer approaches from the Atlantic to the Norwegian Sea.

The fourth difficulty in the application of present dynamic methods to actual conditions in nature has its basis in the fact that Bjerkness’ circulation theorem is valid only in the absence of acceleration or retardation, that is, in the absence of changes in kinetic or potential energy, as explicitly stated by its author (Bjerkness, 1910). The deduction of the velocity equation is also based upon the assumption of absence of friction. It is thus obvious that ideal flow can only occur where and when retardation by friction is exactly balanced by acceleration due to wind or to thermal effects upon the distribution of mass. It seems equally obvious that such a precise equilibrium between accelerating and retarding factors can only be expected to develop in very exceptional cases rather than having their occurrence as the usual event, and that in consequence dynamic contour lines can only under special rather than general circumstances be assumed to parallel the actual current trajectories. In this prerequisite of an absence of change in kinetic or potential energy is further implied that the equations derived from Bjerkness’ circulation theorem are strictly applicable only to a parallel system of dynamic contour lines, since, when a convergence or divergence† of dynamic contour lines occurs, the application of the velocity equation will give the result of showing a positive or negative change in velocities and kinetic energy along the same contour line, thereby proving by its own results that the conditions did not fulfill the requirements for a valid application of the method. Although all these reservations were very clearly stated by Bjerkness at the first introduction of the circulation theorem, they would seem to have received only scant attention in general oceanography, and a glance at any dynamic topographic map will give a clear impression of the relatively great rarity of a strictly parallel system fulfilling even this prerequisite for the assumption of ideal flow alone, if not also and simultaneously fulfilling all the other prerequisites as well. At this point it is very important to distinguish again

* The manner in which the Gulf Stream traverses the dynamic contour lines, and its jet-like emergence towards the British waters, is not only confirmed, but even further emphasized by a comparison between Jacobsen’s presentation of dynamic topography and Wüst’s recent presentation of the current itself (Wüst 1937, fig. 2), which just came to hand after the completion of this article.

† Not to be confused with a true convergence of trajectories with vertical components of motion.
Figure 41. Dynamic topography of the free surface in the North Atlantic according to Jacobsen (1929).

Figure 42. Surface currents in the North Atlantic according to Meyer. Dynamic contour lines of 45, 5 and 0 dyn, cm. relative elevation entered for comparison according to figure 41.
between the approximate determination of states of motion and the effort to trace actual current trajectories, since it seems indicated that if the disturbing influences are only those of retardation or acceleration in the longitudinal direction, without significant interference from any other of the distorting factors discussed in this article, satisfactory results are likely to be

![Figure 43. Qualitative illustration of probable relationship between dynamic contour lines and trajectories in an idealized oceanic circulation system on the northern hemisphere, subject to acceleration in low latitudes and retardation in high latitudes. Contour lines solid, trajectories dot and dash.](image)

obtained by the dynamic method in so far as one is only concerned with approximate velocities and approximate directions of flow at any particular point, while a very great cumulative error would be likely to develop if the method were used for the determination of current trajectories over longer distances.

In a separate report the writer (Parr 1937c) has endeavored to show how the Caribbean Current with a known direction of flow, apparently must
ascend a dynamic slope in its passage through the Carribbean and Cayman Seas, and has ascribed this ascent to the effects of acceleration from above, caused by the steady trade winds of this region, with the suggestion that the case is probably only an illustration of the general principle that acceleration from above must cause an ascent of calculated dynamic gradient, retardation inversely causing descent. When acceleration or retardation proceeds from below due to topographic constriction or expansion, the writer further suggested that the effects should be the opposite of those caused by accelerating or retarding influences from above; acceleration from below resulting in a descent of calculated dynamic elevation, retardation from below in an ascent. This case of the Caribbean Current the writer considers so well established both in qualitative principle and in the actual example, which, due to topographic limitations, is not open to question with regard to the general direction of flow, that no repetition or further elaboration is necessary on these pages.

If, on the other hand, Jacobsen's illustration of the dynamic topography of the North Atlantic is approximately correct, we may find further, and perhaps even better, corroborative evidence in the north equatorial current, which when Jacobsen's and Meyer's maps (Fig. 41 and 42, see contour lines for 45 dyn. cm. relative elevation entered in Fig. 42 according to Fig. 41) are compared would seem to traverse the contour lines more or less at right angles, ascending westward to greater calculated dynamic elevations under the acceleration received from the trade winds.

If we now finally turn to a comparison between the dynamic topography of the sea surface of the North Atlantic as presented by Jacobsen (1929, Fig. 41 in this article) and the empirical surface current map prepared by Meyer (1928, Fig. 42 in this article), we get a convincing impression of the extent of the disagreement between dynamic contour lines and actual current trajectories, seeing that these run almost as frequently at right angles to each other as they run parallel.

In conclusion we might therefore attempt to deduct the general principle underlying this distorted relationship between contour lines and current trajectories. Under real conditions in nature, it is evident that both frictional retardation and acceleration by winds and other factors must influence the actual velocities of flow. In a stable circulation the sum totals of these retarding and accelerating factors over the entire circuit must exactly balance each other. If this was not the case, but total frictional retardation through the whole system exceeded the acceleration by wind, etc., the entire circulation would be running down to a stop. Or if the sum total of accelerating forces exceeded the retarding forces the speed of rotation would be constantly increasing.

If therefore we take an exact balance between total acceleration and retardation in a steady circulation for granted, we are thereby also forced
to accept as an unavoidable conclusion that acceleration and retardation will not balance each other over the greatest parts of the rotation system, since we a priori know that e. g. the accelerating effect of the steady trade winds must far exceed the accelerating effect of the weaker and more variable winds in other regions, and must even compensate for unfavorable winds over portions of the circulation system. It therefore seems evident that excessive acceleration in regions of strong favorable winds must cause an increase in kinetic or potential energy, to be dissipated again by excessive retardation in regions of less favorable wind conditions, and thus maintain the steady balance of the entire circulation system. It also seems fairly clear that the trade-wind belt must be the region of greatest excess acceleration and the northeastern quadrant (in the North Atlantic, at least) will generally be the region of greatest acceleration deficiency (see also Montgomery, 1936). Simplifying still further, one might say that the equatorial half of the rotation system will tend to be accelerated, the polar half retarded.

Let us now assume that we start with an ideal system of rotary flow, with equally spaced concentric dynamic contour lines also representing actual trajectories. Let the southern half (on the Northern Hemisphere) be subject to acceleration in excess of retardation. According to our previous conclusions (Parr 1937c, see p. 130 above) we would then expect the actual trajectories to begin an ascent of the original dynamic gradient. But by this ascent the current will also bring in towards higher dynamic elevations some of the water masses whose original distribution determined the concentric configuration of the dynamic contour lines. In other words, the originally concentric contours will also, under the influence of acceleration, have been drawn in towards the center of rotation of the entire system. Since the premise that the accelerated current must ascend, however, still holds, the deflection of the dynamic contour lines towards the center of rotation must be less than the deflection of the trajectories in a stable relationship between acceleration and dynamic topography, as shown at the lower left (A) in Fig. 43. It will hardly be necessary to elaborate the inverse reasoning for the descent of trajectories, and outward deflection of contour lines in the retarded half of the system.

If we now trace trajectories and deflected contour lines from common points of origin along the boundary between acceleration and retardation in the right half of the system, we get the picture shown in Fig. 43, in which the dynamic contour lines may form an excentric system of circles or of relatively broad oval forms, and the trajectories a similar system of narrower oblong curves intercepting the contour lines towards the center of circulation under acceleration and away from the center by net retardation.

Comparing Fig. 41 and Fig. 42, we find exactly the same general relationship between the dynamic contour line system of the North Atlantic, cal-
culated from actual observations, and the surface trajectories determined from empirical data, which are further overwhelmingly confirmed by the distributions of surface salinities and temperatures (allowing for cooling and mixing) (see Schott 1926, Pl. VIII and X) and by biological evidence from the distribution of eel larvae (Schmidt 1933, see Parr 1936a). When we also take topographic influences and the influence of cooling, etc. (Parr 1936a, see p. 126) into account, the agreement in general principle between the abstract Fig. 43 and the empirical Figs. 41 and 42 is so good that it seems quite warranted to offer this qualitative hypothesis of the effects of differentially distributed acceleration and retardation as a tentative basic explanation for the general relationship and distortion between dynamic contour lines and actual trajectories of flow in the major oceanic circulation systems.

LITERATURE REFERRED TO

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